The Theory of Interest

An Undergraduate Introduction to Financial Mathematics

J. Robert Buchanan

2014

Simple Interest (1 of 2)

Definition

Interest is money paid by a bank or other financial institution to an investor or depositor in exchange for the use of the depositor's money.

Amount of interest is (usually) a fraction (called the **interest rate**) of the initial amount deposited called the **principal amount**.

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Amount of interest is (usually) a fraction (called the **interest rate**) of the initial amount deposited called the **principal amount**.

Remark: a bank whose interest rate for depositors is the same as its interest rate for borrowers is called an **ideal bank**.

Simple Interest (2 of 2)

Notation:

r: interest rate per unit time

P: principal amount

A: amount due (account balance)

t: time

These quantities are related through the equation:

$$A = P(1 + rt).$$

Compound Interest (1 of 2)

Once credited to the investor, the interest may be kept by the investor, and may earn interest itself.

If interest is credited once per year, then after t years the amount due is

$$A = P(1 + r)^t.$$

Compound Interest (2 of 2)

If a portion of the interest is credited after a fraction of a year, then the interest is said to be **compounded**.

If there are n **compounding periods** per year, then in t years the amount due is

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

Examples (1 of 2)

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Suppose an account earns 5.75% annually compounded monthly. If the principal amount is \$3104 what is the amount due after three and one-half years?

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Solution:

$$\begin{aligned} A &= P\left(1 + \frac{r}{n}\right)^{tn} \\ &= 3104\left(1 + \frac{0.0575}{12}\right)^{(3.5)(12)} \\ &\approx 3794.15 \end{aligned}$$

Examples (2 of 2)

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Solution:

$$\begin{aligned} A &= P(1 + rt) \\ &= 3104(1 + 0.0575(3.5)) \\ &\approx 3728.68 \end{aligned}$$

Effective Interest Rate

Definition

The annual interest rate equivalent to a given compound interest rate is called the **effective interest rate.**

$$r_e = \left(1 + \frac{r}{n}\right)^n - 1$$

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Remark: the effective interest rate is also known as the **effective yield** or simply as the **yield.**

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$$\begin{aligned} r_e &= \left(1 + \frac{r}{n}\right)^n - 1 \\ &= \left(1 + \frac{0.0575}{12}\right)^{12} - 1 \\ &\approx 0.0590398 \end{aligned}$$

Continuous Compounding

What happens as we increase the frequency of compounding?

$$A = \lim_{n\to\infty} P\left(1+\frac{r}{n}\right)^{nt}$$

Evaluate the limit using l'Hôpital's Rule.

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Evaluate the limit using l'Hôpital's Rule.

Definition

The amount due for **continuously compounded interest** is

$$A = P\,e^{r\,t}$$

Example (1 of 2)

Suppose \$3585 is deposited in an account which pays interest at an annual rate of 6.15% compounded continuously.

1. Find the amount due after two and one half years.
2. Find the equivalent annual effective simple interest rate.

Example (2 of 2)

1. Amount due:

$$A = Pe^{rt} = 3585e^{0.0615(2.5)} \approx 4180.82$$

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2. Effective rate: since $\lim_{n\to\infty}\left(1+\frac{r}{n}\right)^n - 1 = e^r - 1$ then

$$\begin{aligned} r_e &= e^r - 1 \\ &= e^{0.0615} - 1 \\ &\approx 0.0634305 \end{aligned}$$

Present Value

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P &= A\left(1+\frac{r}{n}\right)^{-nt} \quad \text{(discrete compounding)} \\
P &= A\,e^{-rt} \quad \text{(continuous compounding)}
\end{aligned}
$$

Example (1 of 2)

Suppose an investor will receive payments at the end of the next six years in the amounts shown in the table.

Year	1	2	3	4	5	6
Payment	465	233	632	365	334	248

If the interest rate is 3.99% compounded monthly, what is the total present value of the investments?

Example (2 of 2)

Solution:

$$\begin{aligned} P &= \sum_{t=1}^{6} \left(A_t \left(1 + \frac{0.0399}{12} \right)^{-12t} \right) \\ &= \sum_{t=1}^{6} A_t (0.67536)^t \\ &\approx 2003.01 \end{aligned}$$

Example: Lottery

A lottery has a grand prize of \$10M which is paid in ten payments of \$1M annually with the first payment made immediately. If the prevailing annual interest rate is 3.5% compounded monthly, find the present value of the lottery's grand prize.

Equivalence of Cash Flow Streams

The cash flow streams $\mathbf{x} = \{x_0, x_1, \ldots, x_n\}$ and $\mathbf{y} = \{y_0, y_1, \ldots, y_n\}$ are **equivalent** for an ideal bank if and only if the present values of the two streams are equal.

Example: Harvesting a Crop

Suppose you can stock a pond with fish that you can later sell for food. Stocking the pond requires an initial outlay of capital, but once stocked the fish and pond are self-sustaining. You can choose when the harvest the fish, but the longer you wait to harvest, the larger the fish will be. The annually compounded interest rate is 5%. If you harvest after one year the cash flow stream is $\{-100, 200\}$. If you harvest after two years the cash flow stream is $\{-100, 0, 250\}$. When should you harvest?

Geometric Series

Theorem

If $a \neq 1$ *then* $S = 1 + a + a^2 + \cdots + a^n = \dfrac{1 - a^{n+1}}{1 - a}$.

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Proof.

Let $S = 1 + a + a^2 + \cdots + a^n$ then $aS = a + a^2 + \cdots + a^n + a^{n+1}$ and

$$\begin{aligned} S - aS &= (1 + a + \cdots + a^n) - (a + a^2 + \cdots + a^{n+1}) \\ S(1 - a) &= 1 - a^{n+1} \\ S &= \frac{1 - a^{n+1}}{1 - a} \end{aligned}$$

□

Loan Payments (1 of 2)

Suppose a loan of amount P will be paid back discretely (n times per year) over t years. All payments will be the same amount. The unpaid portion of the loan is charged interest at annual rate r compounded n times per year. What is the discrete, regular payment x?

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Hint: the present value of all the payments should equal the amount borrowed.

If the first payment must be made at the end of the first compounding period, then the present value of all the payments is

$$\begin{aligned} & x(1+\frac{r}{n})^{-1} + x(1+\frac{r}{n})^{-2} + \cdots + x(1+\frac{r}{n})^{-nt} \\ = & \quad x(1+\frac{r}{n})^{-1}\frac{1-(1+\frac{r}{n})^{-nt}}{1-(1+\frac{r}{n})^{-1}} \\ = & \quad x\frac{1-(1+\frac{r}{n})^{-nt}}{\frac{r}{n}} \end{aligned}$$

Thus

$$P = x\,\frac{n}{r}\left(1-\left[1+\frac{r}{n}\right]^{-nt}\right)$$

Example

If a person borrows $25,000 for five years at an interest rate of 4.99% compounded monthly and makes equal monthly payments, what is the monthly payment?

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Solution:

$$\begin{aligned} x &= P\frac{r}{n}\left(1 - \left[1 + \frac{r}{n}\right]^{-nt}\right)^{-1} \\ &= 25000\left(\frac{0.0499}{12}\right)\left(1 - \left[1 + \frac{0.0499}{12}\right]^{-(12)(5)}\right)^{-1} \\ &\approx 471.67 \end{aligned}$$

Retirement Savings (1 of 2)

Example

Suppose a person is 25 years of age now and plans to retire at age 65. For the next 40 years they plan to invest a portion of their monthly income in securities which earn interest at the annual rate of 10% compounded monthly. After retirement the person plans on receiving a monthly payment (an annuity) in the absolute amount of $1500 for 30 years. How much should be set aside monthly for retirement?

Solution: The present value of all funds invested for retirement should equal the present value of all funds taken out during retirement.

$$
\begin{aligned}
x \sum_{i=1}^{480} \left(1 + \frac{0.10}{12}\right)^{-i} &= 1500 \sum_{i=481}^{840} \left(1 + \frac{0.10}{12}\right)^{-i} \\
&= 1500 \left(1 + \frac{0.10}{12}\right)^{-480} \sum_{i=1}^{360} \left(1 + \frac{0.10}{12}\right)^{-i} \\
x &= \frac{1500 \left(1 + \frac{0.10}{12}\right)^{-480} \sum_{i=1}^{360} \left(1 + \frac{0.10}{12}\right)^{-i}}{\sum_{i=1}^{480} \left(1 + \frac{0.10}{12}\right)^{-i}} \\
&\approx 27.03
\end{aligned}
$$

Adjusting for Inflation

Definition

An increase in the amount of money in circulation without a commensurate increase in the amount of available goods is a condition known as **inflation**. Thus relative to the supply of goods, the value of the currency is decreased.

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An increase in the amount of money in circulation without a commensurate increase in the amount of available goods is a condition known as **inflation**. Thus relative to the supply of goods, the value of the currency is decreased.

How does inflation (measured at an annual rate i) affect the value of deposits earning interest?

Inflation-adjusted Interest Rate

- Suppose at the current time one unit of currency will purchase one unit of goods.

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- The difference $(1 + r) - (1 + i) = r - i$ will be the real rate of growth in the unit of currency invested now.
- This return on saving will not be earned until one year from now. The present value of $r - i$ under inflation rate i is

$$r_i = \frac{r - i}{1 + i}.$$

Example (revisited)

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Suppose a person is 25 years of age now and plans to retire at age 65. For the next 40 years they plan to invest a portion of their monthly income in securities which earn interest at the rate of 10% compounded monthly. After retirement the person plans on receiving a monthly payment (an annuity) in the absolute amount of $1500 for 30 years. How much should be set aside monthly for retirement if the annual inflation rate is 3%?

Effects of Inflation

Solution: The inflation adjusted return on saving is

$$r_i = \frac{r - i}{1 + i} = \frac{0.10 - 0.03}{1 + 0.03} \approx 0.0679612.$$

Using this value in place of r in the previous example we have

$$x = \frac{1500\left(1 + \frac{0.0679612}{12}\right)^{-480} \sum_{i=1}^{360}\left(1 + \frac{0.0679612}{12}\right)^{-i}}{\sum_{i=1}^{480}\left(1 + \frac{0.0679612}{12}\right)^{-i}} \approx 92.84.$$

Mortgage Amortization (1 of 4)

Suppose a person takes out a mortgage loan in the amount of L and will make n equal monthly payments of amount x where the annual interest rate is r compounded monthly.

1. Express x as a function of L, r, and n.
2. After the jth month, how much of the original amount borrowed remains?
3. How much of the jth payment goes to interest and how much goes to pay down the amount borrowed?

Mortgage Amortization (2 of 4)

The sum of the present values of all the payments must equal the amount loaned.

$$
\begin{aligned}
L &= \sum_{i=1}^{n} \frac{x}{(1+r/12)^i} \\
&= x(1+r/12)^{-1} \sum_{i=0}^{n-1} (1+r/12)^{-i} \\
&= x(1+r/12)^{-1} \frac{1-(1+r/12)^{-n}}{1-(1+r/12)^{-1}} \\
&= \frac{x\left[1-(1+r/12)^{-n}\right]}{(1+r/12)-1} \\
&= \frac{12x}{r}\left[1-(1+r/12)^{-n}\right]
\end{aligned}
$$

Mortgage Amortization (3 of 4)

The outstanding balance on the loan immediately after the jth monthly payment will be the sum of the present values of the remaining payments. Let L_j denote the outstanding balance immediately after the jth payment, then

$$\begin{aligned}
L_j &= \sum_{i=1}^{n-j} \frac{x}{\left(1+\frac{r}{12}\right)^i} \\
&= x(1+r/12)^{-1} \sum_{i=0}^{n-j-1} \left(1+\frac{r}{12}\right)^{-i} \\
&= x(1+r/12)^{-1} \frac{1-(1+r/12)^{-n+j}}{1-(1+r/12)^{-1}} \\
&= \frac{x\left[1-(1+r/12)^{-n+j}\right]}{(1+r/12)-1} \\
&= \frac{12x}{r}\left[1-\left(1+\frac{r}{12}\right)^{-n+j}\right].
\end{aligned}$$

Mortgage Amortization (4 of 4)

If I_j represents the amount of interest in the jth payment, then

$$I_j = L_{j-1}(r/12) = x\left[1 - \left(1 + \frac{r}{12}\right)^{-n+j-1}\right].$$

The amount of principal repaid in the jth payment is

$$P_j = x - I_j = x\left(1 + \frac{r}{12}\right)^{-n+j-1}.$$

Mortgage Example (1 of 3)

Suppose \$284,000 is borrowed to purchase a house. The annual interest rate of the mortgage is 4.75% compounded monthly and the term of the mortgage is 15 years.

1. What is the regular monthly payment?
2. What is the balance on the outstanding principal after the 99th payment?
3. How much of the 100th payment goes to pay interest?
4. How much of the 100th payment goes to repay principal?

Mortgage Example (2 of 3)

1. What is the regular monthly payment?

2. What is the balance on the outstanding principal after the 99th payment?

Mortgage Example (2 of 3)

1. What is the regular monthly payment?

$$x = \frac{L\,r/12}{1 - \left(\frac{12}{12+r}\right)^n} = \frac{284000(0.0475/12)}{1 - \left(\frac{12}{12.0475}\right)^{180}} = \$2,209.04$$

2. What is the balance on the outstanding principal after the 99th payment?

Mortgage Example (2 of 3)

1. What is the regular monthly payment?

$$x = \frac{Lr/12}{1 - \left(\frac{12}{12+r}\right)^n} = \frac{284000(0.0475/12)}{1 - \left(\frac{12}{12.0475}\right)^{180}} = \$2,209.04$$

2. What is the balance on the outstanding principal after the 99th payment?

$$L_{99} = \frac{12(2209.04)}{0.0475}\left[1 - \left(\frac{12}{12.0475}\right)^{180-99}\right] = \$152,825.70$$

Mortgage Example (3 of 3)

3. How much of the 100th payment goes to pay interest?

4. How much of the 100th payment goes to repay principal?

Mortgage Example (3 of 3)

3. How much of the 100th payment goes to pay interest?

$$I_{100} = (152825.70)(0.0475/12) = \$604.94$$

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Mortgage Example (3 of 3)

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$$I_{100} = (152825.70)(0.0475/12) = \$604.94$$

4. How much of the 100th payment goes to repay principal?

$$P_{100} = 2209.04 - 604.94 = \$1,604.10$$

Continuously Varying Interest Rates (1 of 2)

Definition

If interest is compounded continuously at a time-dependent rate $r(t)$, the function $r(t)$ is referred to as the **spot rate**.

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Continuously Varying Interest Rates (1 of 2)

Definition

If interest is compounded continuously at a time-dependent rate $r(t)$, the function $r(t)$ is referred to as the **spot rate.**

- Suppose the amount due at $t = 0$ is $A(0) = 1$.
- The amount due at time t is $A(t)$ and if Δt is small then

$$\begin{aligned} A(t + \Delta t) &\approx A(t)(1 + r(t)\Delta t) \\ \frac{A(t + \Delta t) - A(t)}{\Delta t} &\approx r(t)A(t) \\ A'(t) &= r(t)A(t). \end{aligned}$$

Continuously Varying Interest Rates (2 of 2)

Amount due at time $t > 0$ on a unit deposit:

$$A(t) = e^{\int_0^t r(s)\,ds}$$

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Definition

The average of the spot rate over the interval $[0, t]$

$$\bar{r}(t) = \frac{1}{t}\int_0^t r(s)\,ds$$

is called the **yield curve.**

Example (1 of 3)

Suppose the spot rate is $r(t) = \dfrac{r_1}{1+t} + \dfrac{r_2 t}{1+t}$.

1. Find the yield curve $\bar{r}(t)$.
2. Find the present value of a unit due at time $t > 0$.

Example (2 of 3)

Yield curve:

$$\begin{aligned}\bar{r}(t) &= \frac{1}{t}\int_0^t \left(\frac{r_1}{1+s} + \frac{r_2 s}{1+s}\right) ds \\ &= \frac{r_1}{t}\ln(1+t) + \frac{r_2}{t}\left(t - \ln(1+t)\right) \\ &= r_2 + \frac{r_1 - r_2}{t}\ln(1+t)\end{aligned}$$

Present value of a unit amount:

$$\begin{aligned} P(t) &= e^{-\int_0^t r(s)\,ds} \\ &= e^{-t\bar{r}(t)} \\ &= e^{-t\left(r_2+\frac{r_1-r_2}{t}\ln(1+t)\right)} \\ &= e^{-r_2 t-(r_1-r_2)\ln(1+t)} \\ &= (1+t)^{r_2-r_1}e^{-r_2 t} \end{aligned}$$

Rate of Return

Definition

If an investment of amount P now receives an amount due of A one time unit from now, the **rate of return** (denoted r) is the equivalent interest rate so that the present value of A is P.

$$P = A(1 + r)^{-1}$$

Example

If you loan a friend \$100 today with the understanding that they will pay you back \$110 in one year's time, what is the rate of return?

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Solution:

$$\begin{aligned} P &= A(1+r)^{-1} \\ 100 &= 110(1+r)^{-1} \\ 1+r &= \frac{110}{100} \\ r &= 0.10 \end{aligned}$$

General Setting

Suppose you invest an amount P now and receive a sequence of positive payoffs $\{A_1, A_2, \ldots, A_n\}$ at regular intervals. The rate of return per period is the interest rate such that the present value of the sequence of payoffs is equal to the amount invested.

$$P = \sum_{i=1}^{n} A_i (1 + r)^{-i}.$$

Example

Suppose you loan a friend \$100 with the agreement that they will pay you at the end of each year for the next five years amounts $\{21, 22, 23, 24, 25\}$. Find the annual rate of return.

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Suppose you loan a friend \$100 with the agreement that they will pay you at the end of each year for the next five years amounts $\{21, 22, 23, 24, 25\}$. Find the annual rate of return.

Solution:

$$\begin{aligned} 100 &= \frac{21}{1+r} + \frac{22}{(1+r)^2} + \frac{23}{(1+r)^3} + \frac{24}{(1+r)^4} + \frac{25}{(1+r)^5} \\ r &\approx 0.0470299 \end{aligned}$$

The solution to the equation is approximated using Newton's method with an initial approximation of 0.03.

Example: Harvesting a Crop

Suppose you can stock a pond with fish that you can later sell for food. Stocking the pond requires an initial outlay of capital, but once stocked the fish and pond are self-sustaining. You can choose when the harvest the fish, but the longer you wait to harvest, the larger the fish will be. The annually compounded interest rate is 5%. If you harvest after one year the cash flow stream is $\{-100, 200\}$. If you harvest after two years the cash flow stream is $\{-100, 0, 250\}$. Using the rate of return as the basis for the decision, when should you harvest?

Continuous Income Streams

Suppose the income received per unit time is the function $S(t)$ for $a \leq t \leq b$.

A **Riemann sum** approximates the total income received

$$\sum_{k=1}^{n} S(t_k)(t_k - t_{k-1}).$$

As $n \to \infty$ the total income is

$$S_{\text{tot}} = \int_a^b S(t)\,dt.$$

Amount Due and Present Value

If the continuously compounded interest rate is $r(t)$, the present value at time $t = 0$ of the income stream $S(t)$ for $0 \leq t \leq T$ is

$$P = \int_0^T e^{-r(t)\,t} S(t)\,dt.$$

The future value at $t = T$ of the income stream is

$$A = \int_0^T e^{r(t)(T-t)} S(t)\,dt.$$

Example

Suppose the slot machine floor of a new casino is expected to bring in $\$30,000$ per day. What is the present value of the first year's slot machine revenue assuming the continuously compounded annual interest rate is 3.55%?

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$$
\begin{aligned}
P &= \int_0^1 (30000)(365)e^{-0.0355t}\,dt \\
&= \frac{(30000)(365)}{-0.0355} e^{-0.0355t}\Big|_0^1 \approx \$10,757,917.19
\end{aligned}
$$

Credits

These slides are adapted from the textbook,

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