

Applied Mathematics

J. Robert Buchanan

September 27, 2003

Millersville University of Pennsylvania

email: Bob.Buchanan@millersville.edu

Topics

- Allometry, shape, and form
- Pharmacokinetic modeling
- Chemical reaction dynamics
- Population biology and mathematical ecology
- Traffic dynamics
- Optimal control and utility
- Elections and voting
- Games and chance and skill
- Birth and death

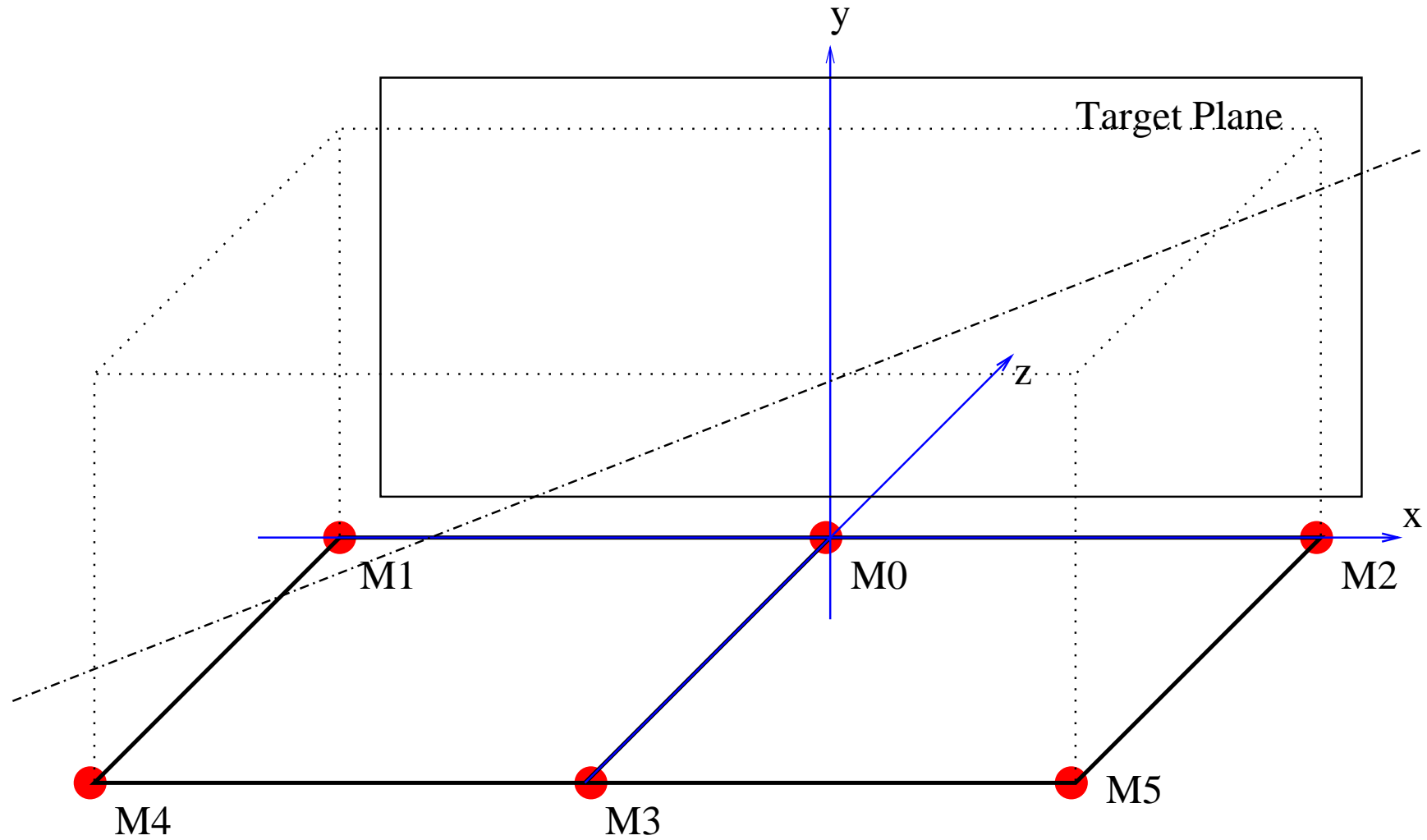
Techniques

- Axiomatic systems
- Geometry and trigonometry
- 1, 2, and 3-dim differential equations
- Numerical simulations
- Parameter estimation and curve fitting
- Graph theory
- Dynamical systems
- Conservation laws
- Control theory
- Elementary probability

Careers for Applied Mathematicians

- LOMAH (Location of Miss and Hit) Six microphones arranged in a rectangular array
- Plywood target plane centered and aligned vertically and perpendicular to microphone array.
- Microphones record the time of arrival of the shockwave generated by supersonic projectile.

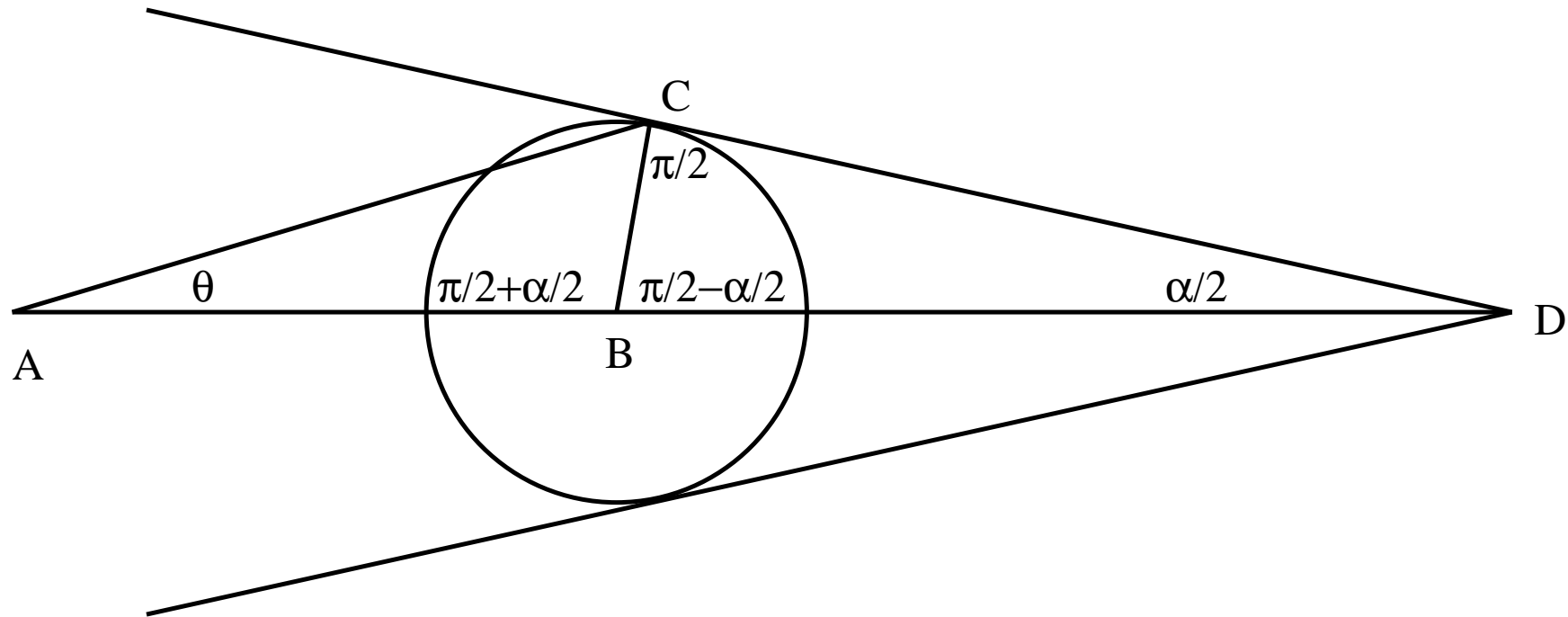
LOMAH



Acoustic Measuring Principle

- Projectile traveling faster than speed of sound creates conical shock wave.
- Opening angle α of shock cone is determined by ratio of projectile velocity $\|\vec{v}\|$ to sound velocity c .
- Cone propagates perpendicularly to its surface at speed of sound c .
- Axis of cone is trajectory of projectile.
- Intersection of cone axis with target plane is hit location.

Shock Cone



Traveltime from A to D is the sum of the traveltimes from A to B and B to C.

Financial Mathematics

- **Arbitrage:** a trading strategy which takes advantage of two securities being mispriced relative to one another in order to make a profit.
- **Options:** the right, but not the obligation, to purchase or sell a security at an agreed upon price in the future.
- **Volatility:** the range of movement in the price of a security
- **Black-Scholes Pricing Formula:** a mathematical formula developed by Fischer Black and Myron Scholes (and extended by Robert Merton) for pricing options. They won the Nobel Prize in Economics in 1997 for this work.

Options and Arbitrage

- **Call:** an option which allows the owner to buy a security in the future at a guaranteed price. The symbol C will denote the price of a call option.
- **Put:** an option (with price P) which allows the owner to sell a security in the future at a guaranteed price.
- **Strike price:** the future guaranteed price (K) of the security for the owner of an option.
- **Expiration time:** the future date (T) by which an option must be exercised or it is lost.
 - **European options:** exercised only when $t = T$.
 - **American options:** exercised whenever $0 \leq t \leq T$.

European Put-Call Parity

There exists a relationship between the price of a security S , the prices of calls C and puts P with the same strike price K and exercise time T , and the prevailing risk-free interest rate r .

$$S + P_e = C_e + Ke^{-rT}$$

If this relationship does not hold, then there is a risk-free way to make a guaranteed profit with no personal investment.

The following two examples suggest a means by which this formula is proven.

Example 1

Suppose $S + P_e > C_e + Ke^{-rT}$.

Let $S = 31$, $K = 30$, $C_e = 3$, $P_e = 2.25$, $r = 10\%$, and $T = 0.25$. Then

$$\begin{aligned}S + P_e &= 33.25 \\C_e + Ke^{-rT} &= 32.26\end{aligned}$$

1. Buy the Call and sell short the security and the Put.
This would generate in cash $S + P_e - C_e = 30.25$.
2. Invest our cash for the life of the option in the bank.
After 3 months we have 31.02 in the bank.
3. At the exercise time we buy the security at the strike price and walk away with a profit of $31.02 - 30 = 1.02$.

Example 2

Suppose $S + P_e < C_e + Ke^{-rT}$.

Let $S = 31$, $K = 30$, $C_e = 3$, $P_e = 1$, $r = 10\%$, and $T = 0.25$.

Then

$$\begin{aligned}S + P_e &= 32 \\C_e + Ke^{-rT} &= 32.26\end{aligned}$$

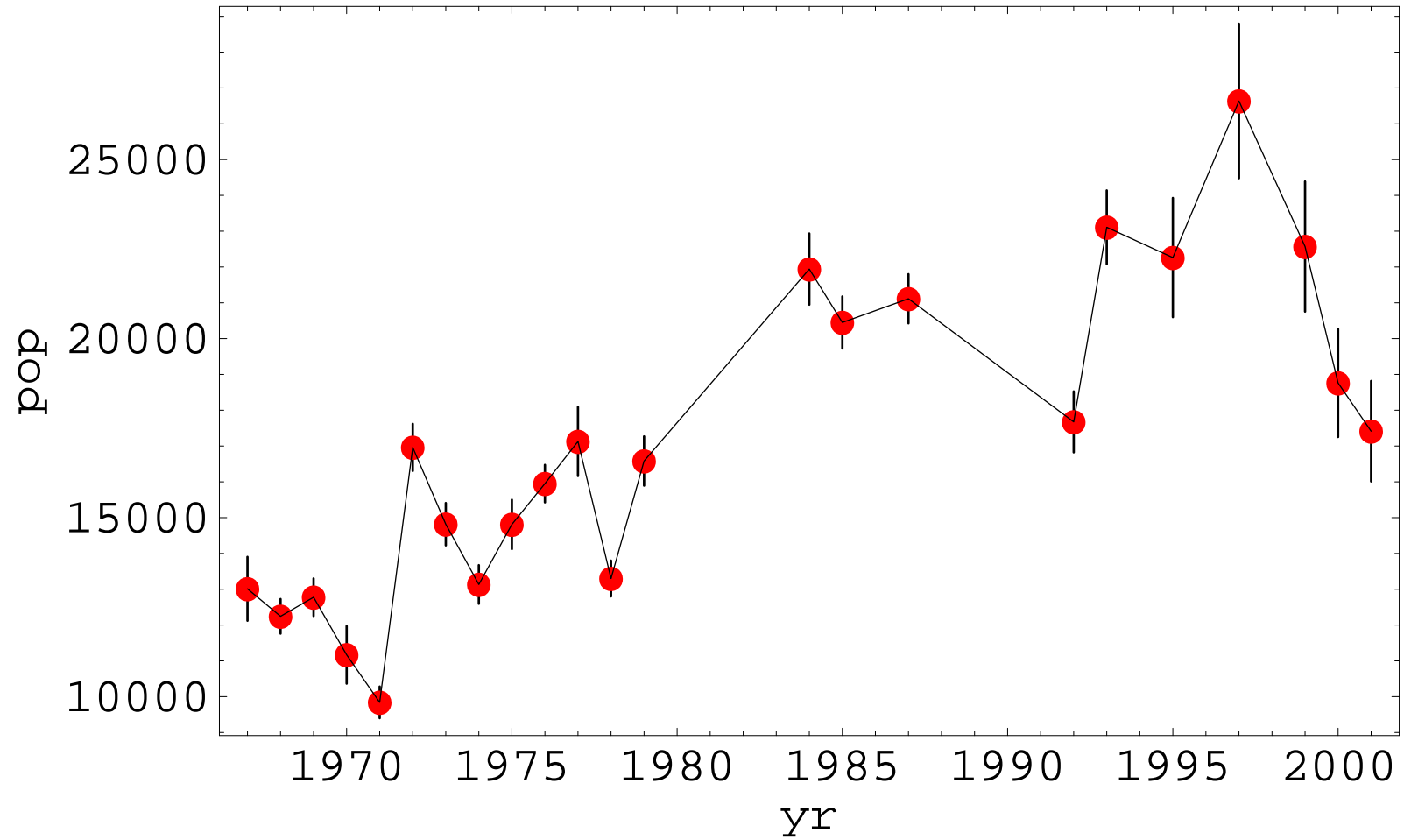
1. Buy the security and the Put and sell short the Call.
This would require that we borrow $S + P_e - C_e = 29$.
2. After 3 months we owe the bank 29.73.
3. At the exercise time we sell the security at the strike price and walk away with a profit of $30 - 29.73 = 0.27$.

Mathematical Ecology

- Gray Whale, Latin name *Eschrichtius robustus*, length 40–50 feet (12–15 m), weight 50,000–80,000 lbs (23,000-36,000 kg), lifespan of up to 50 years.



Census



Recovery?

Q: Has the stock of eastern Pacific gray whales recovered sufficiently that regulatory protection is no longer needed?

- Yearly census results continue to oscillate
- Mathematical models exhibit a decrease in population during 1968–1988 while census numbers showed an *increase*
- Models do not exhibit depletion of stock by 1900

Q: Can the behavior of mathematical models be reconciled with the census data?

Homeland Security

- **Data Mining and Pattern Recognition** - discovering patterns, structure, or associations in large bodies of empirical data.
- **Detection and Epidemiology of Bio-Terrorist Attacks** - how will we distinguish a malicious attack from a naturally occurring outbreak?
- **Voice and Image Recognition** - identifying key telephone conversations among the millions that take place every day.
- **Communications and Computer Security** - securing communications with encryption, protecting networks from disruptive attacks, analyzing network traffic.
- **Data Fusion** - synthesize information from diverse sources to make prudent decisions.