

A Network Model of Alcoholism and Alcohol Policy

J Robert Buchanan

Millersville University of Pennsylvania

email: Bob.Buchanan@millersville.edu

Acknowledgments

R. Braun ^a, P. Broadbridge^a, E. Faulkner ^b, A. Feist ^c, M. Ferrara ^d, A. Gadbois ^e, J. Gleeson ^f, M. Gratton^c, F. Lazebnik^a, J. Pelesko^a, Rakesh^a, Z. Sun ^g, A. Ursan ^h, R. Wilson^a, N. Zhang^d, X. Zhang^a

^aUniversity of Delaware

^bQuantum Leap Innovations

^cDuke University

^dRensselaer Polytechnic Institute

^eMcGill University

^fUniversity College Cork

^gUniversity of Maryland, Baltimore County

^hWorcester Polytechnic Institute

Vodka and Tonic

- **Ingredients:**
 - 2 oz. vodka
 - 2 oz. tonic water
- Add chilled vodka to tonic in a glass containing ice. Garnish with a wedge of lime.



MPI2004: Problem Statement

- Effect of social structures on the spread and persistence of alcoholism and alcohol-related problems
- Effect of treatment policies on alcoholism and alcohol-related problems
- Feasibility of incorporating survey data ($N = 2627$) on the incidence of alcoholism in Delaware by zip code.

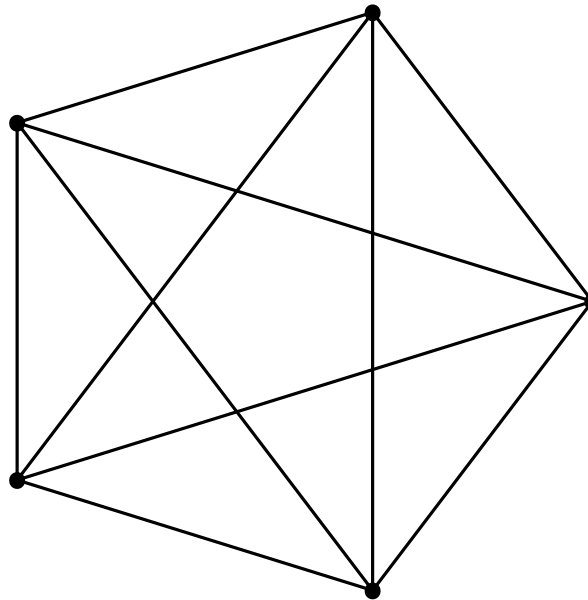
Will not investigate the causes of alcoholism or its related problems.

Modeling Approach

Social interactions are modeled using a **graph** or **network**.

Vertices: correspond to individual persons

Edges: connect two vertices if those people know each other in any context in which they may drink together (e.g. friends, co-workers, family).



Graph Theory Glossary

Degree: number of edges connected to a vertex

Component: set of vertices that can be reached via connecting edges

Geodesic path: shortest path (possibly non-unique) from one vertex to another

Diameter: number of edges in the longest geodesic path between any two vertices

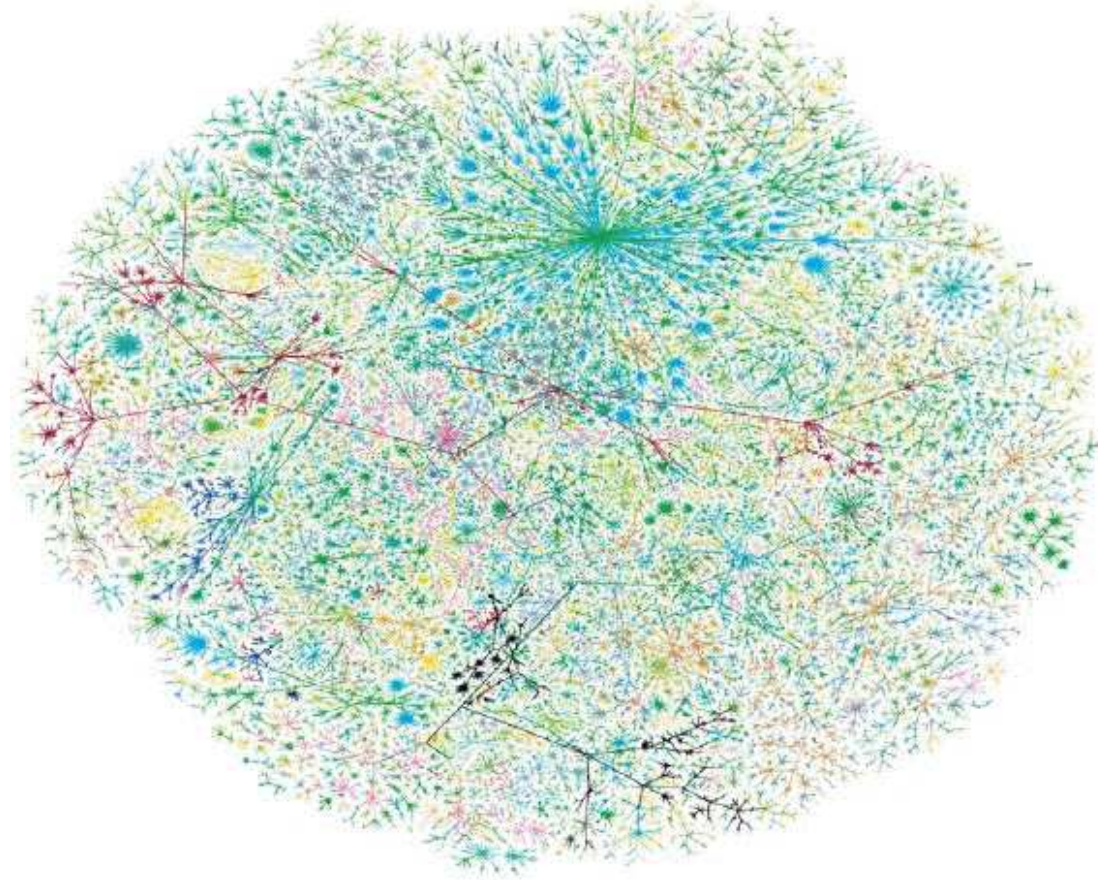
Previous Applications

- Information networks: citation patterns in published research, World Wide Web

Lotka's Law of Scientific Productivity (1926): the number of scientists who have published k papers is proportional to $k^{-\alpha}$, for some constant α .

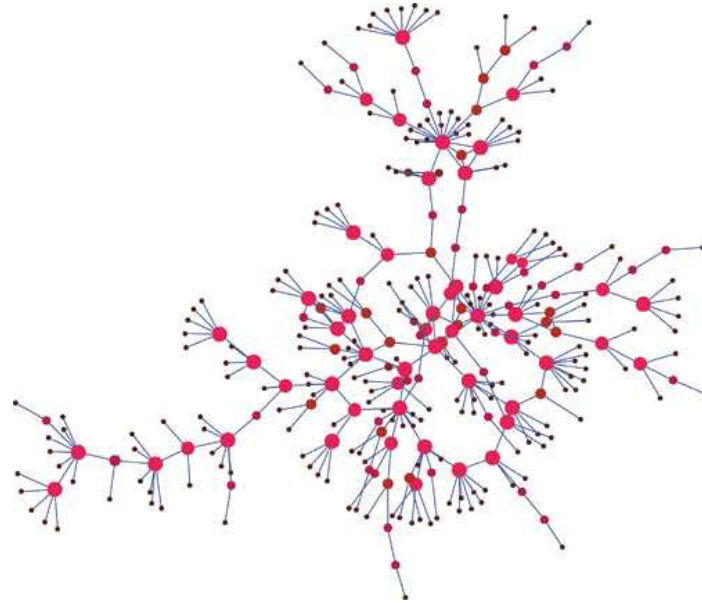
- Technological networks: electricity grid, airline routes, Internet
- Biological networks: metabolic pathways, genetic regulation and expression, food chains

Internet



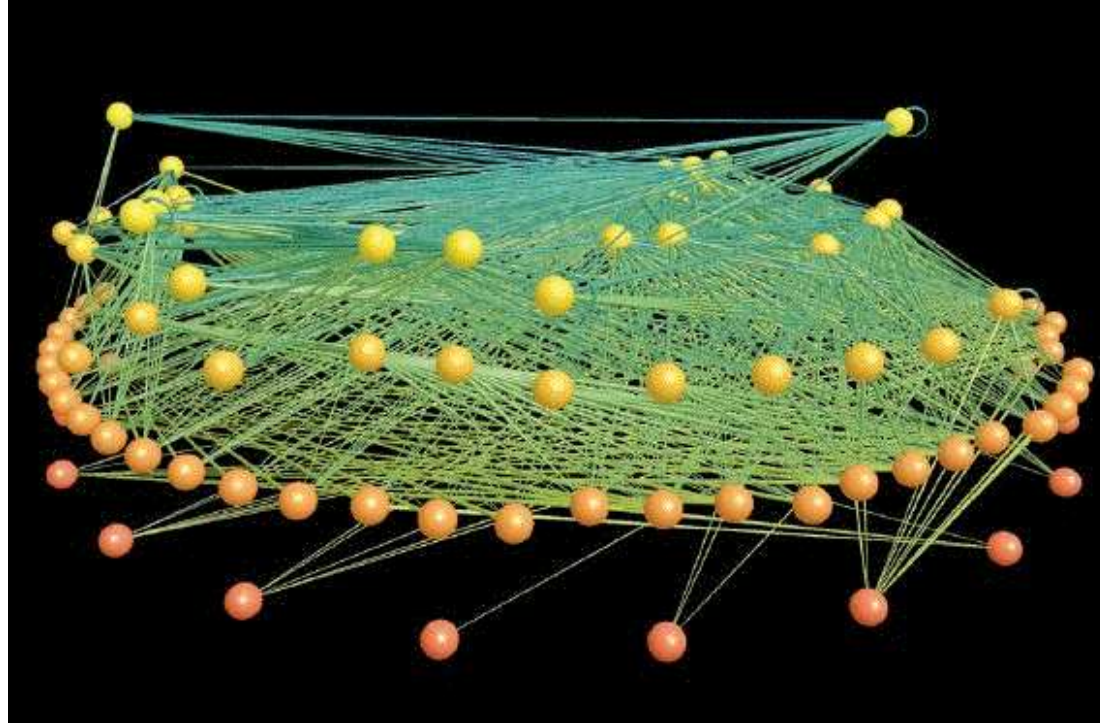
Structure of the Internet (Hal Burch and Bill Cheswick, Lumeta Corporation).

Sexual Contacts



Social network (J.J. Potterat, *et al*, "Risk network Structure in the early epidemic phase of HIV transmission in Colorado Springs," *Sexually Transmitted Infections*, 78 (2002), pp. i159-i163).

Food Chain

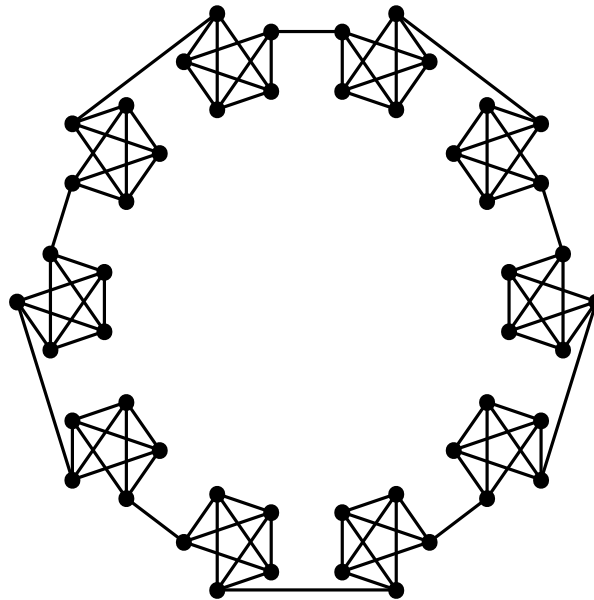


Predator-prey interactions in a freshwater lake (Richard Williams and Neo Martinez. *Ecological Monographs*, 61 (1991), pp. 367-392).

Cavemen and Caves

Assume that society is made up of m caves each containing k cavemen.

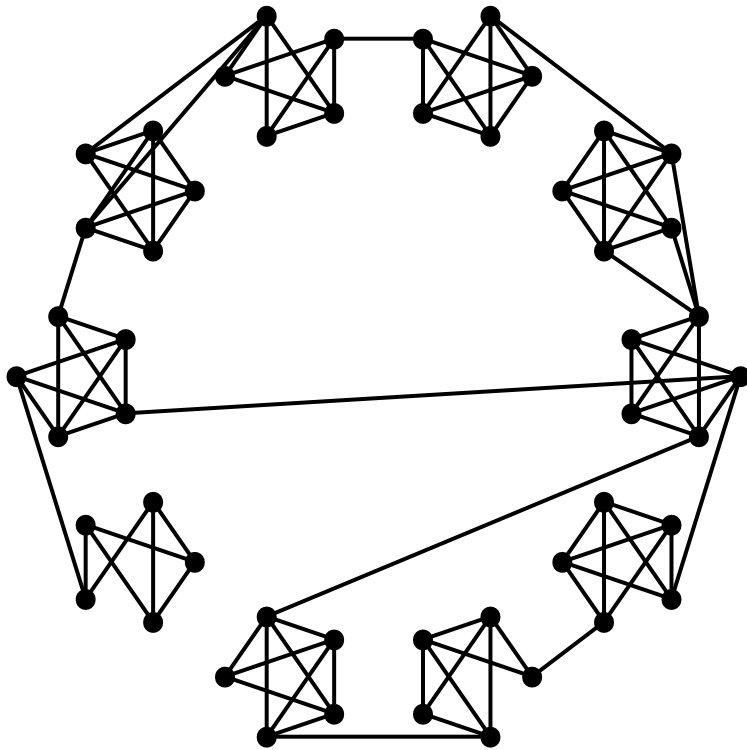
Cave dwellers are tightly connected and caves are loosely connected.



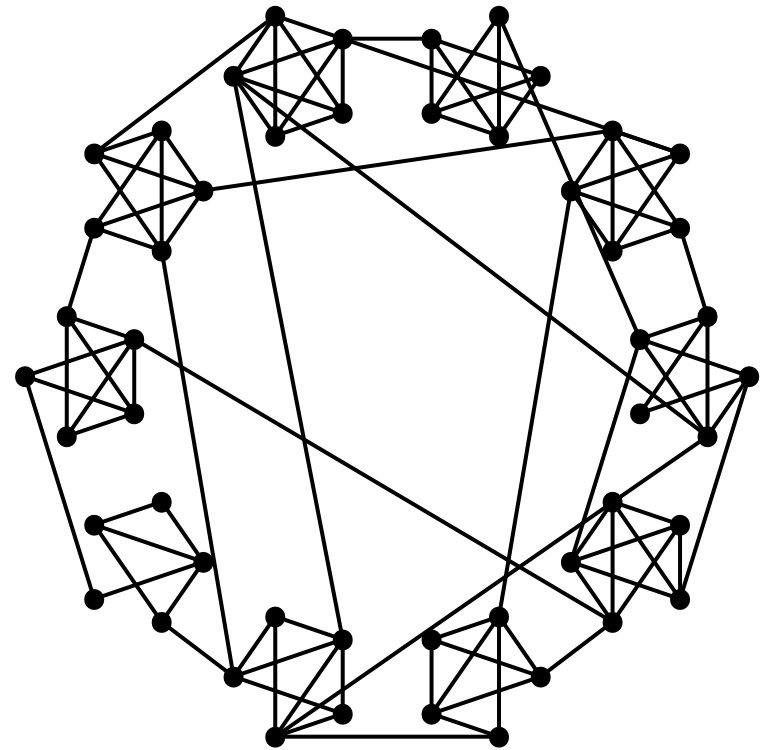
Diameter: 15

Rewiring Caves I

Connected caveman graph is too highly organized.
Randomly reconfigured a proportion of the edges.

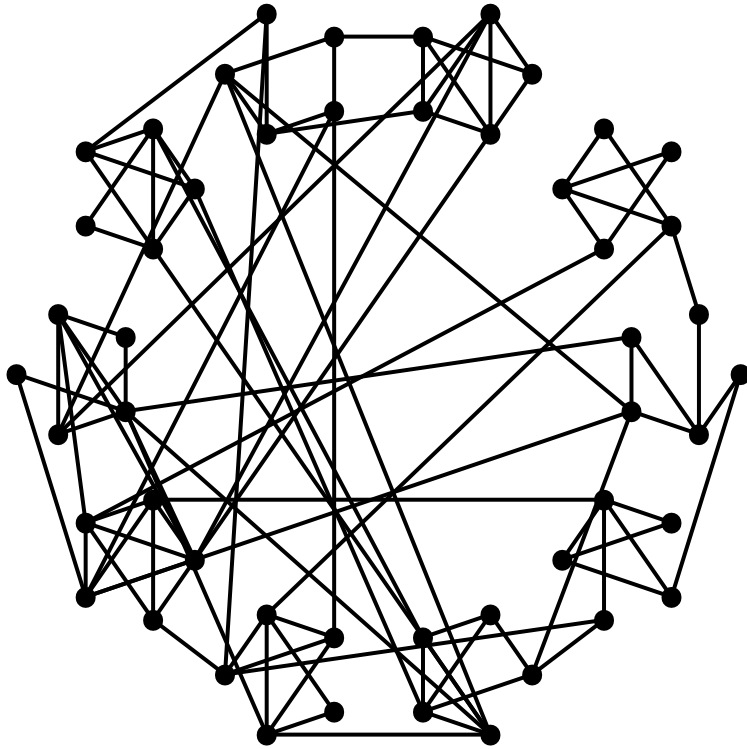


Rewire 5%, Diameter: 6

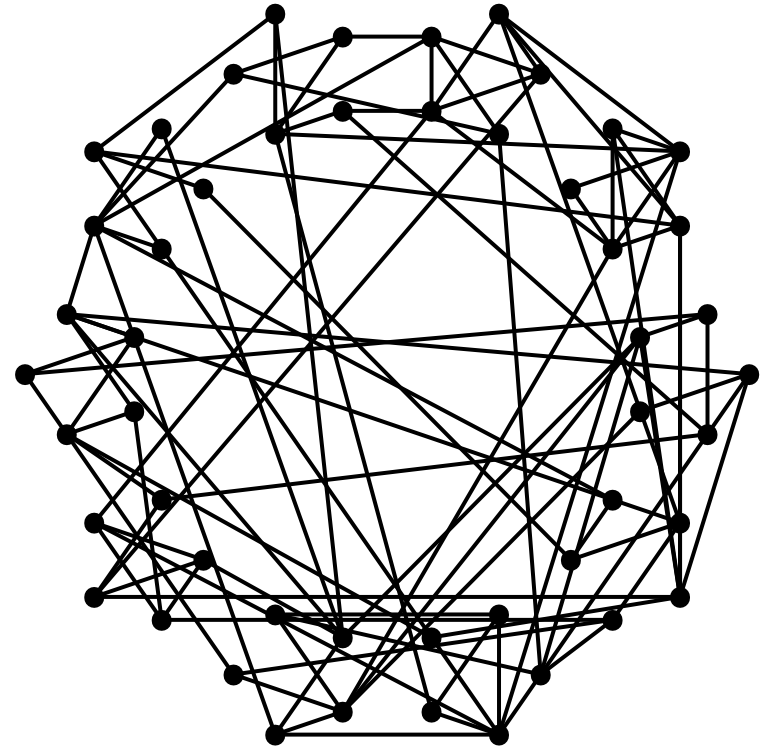


Rewire 10%, Diameter: 6

Rewiring Caves II



Rewire 25%, Diameter: 5



Rewire 50%, Diameter: 4

Small World Effect

Experiment: Given the name of a target individual pass a letter to the target using only first-name acquaintances (Milgram (1967)).

Result: 75% letters lost, 25% delivered with an average path length of 6 edges.

Pop culture: Erdős numbers, Bacon numbers

1. Robert Buchanan was in *Gregory's Girl* (1981) with John Gordon Sinclair,
2. John Gordon Sinclair was in *Erik the Viking* (1989) with Tim Robbins,
3. Tim Robbins was in *Mystic River* (2003) with Kevin Bacon.

Blue Martini

- **Ingredients:**
 - 1 shot vodka
 - 1 shot tequila
 - 1.5 shots blue Curaçao
 - lemonade
- Mix vodka, tequila, and Curaçao with ice in a shaker.
- Strain mixture into a martini glass containing ice, add a splash of lemonade.



Model Parameters

Associated with each vertex ...

alcoholism tendency: continuous parameter value $v \in [0, 1]$, 0 denotes non-drinker, 1 denotes alcoholic.

resiliency: continuous parameter value $r \in [0, 1]$, 0 denotes spineless jellyfish, 1 denotes impervious to peer pressure.

During simulation initialization ...

- v is randomly chosen from a uniform distribution on $[0, 1]$.
- r is randomly chosen from a normal distribution with $\mu = 0.5$ and $\sigma = 1/6$.

Models

Notation: v_i^j , the alcoholic tendency of vertex i at time j .

$$v_i^{j+1} = v_i^j + \lambda v_i^j (1 - v_i^j) (n_i^j - r_i) \quad \text{for } 1 \leq i \leq N, j \geq 1$$

λ convergence parameter, controls how quickly model converges to its asymptotic behavior.

n_i^j average alcoholism tendency of the neighbors of the i^{th} vertex at time j .

Matrix Notation

Let A be the adjacency matrix of the network.

$$A_{ij} = \begin{cases} 1 & \text{if vertex } i \text{ is linked to vertex } j, \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{v}^j = \langle v_1^j, v_2^j, \dots, v_N^j \rangle$$

$$\mathbf{r} = \langle r_1, r_2, \dots, r_N \rangle$$

$$\mathbf{n}^j = \left\langle \frac{(A\mathbf{v}^j)_1}{\text{deg } 1}, \frac{(A\mathbf{v}^j)_2}{\text{deg } 2}, \dots, \frac{(A\mathbf{v}^j)_N}{\text{deg } N} \right\rangle$$

$$\mathbf{v}^{j+1} = \mathbf{v}^j + \lambda \mathbf{v}^j * (\mathbf{1} - \mathbf{v}^j) * (\mathbf{n}^j - \mathbf{r})$$

Element-wise multiplication is denoted by $*$.

Dynamics

Equilibria:

$\mathbf{v} = \mathbf{0}$: alcoholism does not arise in a society where no one drinks

$\mathbf{v} = \mathbf{1}$: temperance does not arise in a society of alcoholics

$v_i \in \{0, 1\}$ for $i = 1, \dots, N$: non-drinkers and alcoholics are unaffected by their peers

$\mathbf{v} = \mathbf{n}$: tipping point

There may be as many as 3^N equilibria for the model.

Stability and Instability

Theorem 0.1 Consider an adjacency matrix A , a vector of resiliencies \mathbf{r} , and a vector of vertex states \mathbf{v} in which $v_i \in \{0, 1\}$. Suppose that $n_i^j \neq r_i$ for all $i = 1, \dots, N$. The vertex state \mathbf{v} is a stable equilibrium of the model if and only if $H(n_i^j - r_i) = v_i$ for all i , where H is the Heaviside function.

$$v_i^{j+1} = v_i^j + \lambda v_i^j (1 - v_i^j) (n_i^j - r_i)$$

Theorem 0.2 Consider an adjacency matrix A , a vector of resiliencies \mathbf{r} , and a vector of vertex states \mathbf{v} in which $n_i = r_i$ and $0 < v_i < 1$ for all $i = 1, \dots, N$. The vertex state \mathbf{v} is an unstable equilibrium of the cubic model.

Tipping Behavior

Corollary 0.3 Consider an adjacency matrix A , a vector of resiliencies \mathbf{r} , and a vector of vertex states \mathbf{v} in which $n_i = r_i$ and $v_i \neq 0$ for all $i = 1, \dots, N$. Then if there exists an index k for which $v_k < 1$ then any perturbation which increases v_k leads to an asymptotic network state of $\mathbf{v}^\infty = \mathbf{1}$.

Corollary 0.4 Consider an adjacency matrix A , a vector of resiliencies \mathbf{r} , and a vector of vertex states \mathbf{v} in which $n_i = r_i$ and $v_i \neq 1$ for all $i = 1, \dots, N$. Then if there exists an index k for which $0 < v_k$ then any perturbation which decreases v_k leads to an asymptotic network state of $\mathbf{v}^\infty = \mathbf{0}$.

Apple Martini

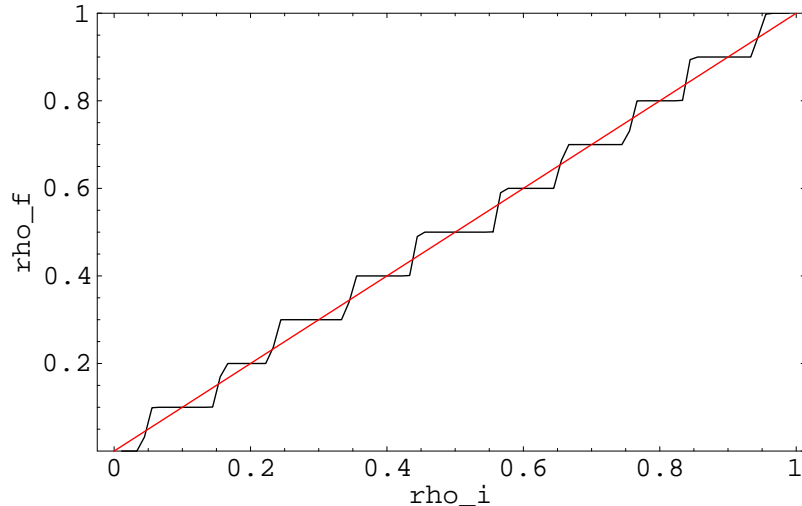
- **Ingredients:**
 - 1.5 shots vanilla vodka
 - 1.5 shots apple schnapps
 - sweet and sour mix
- Mix vodka and schnapps with ice in a shaker
- Strain mixture into martini glass, add splash of sweet and sour mix
- Garnish with thinly sliced granny smith apple



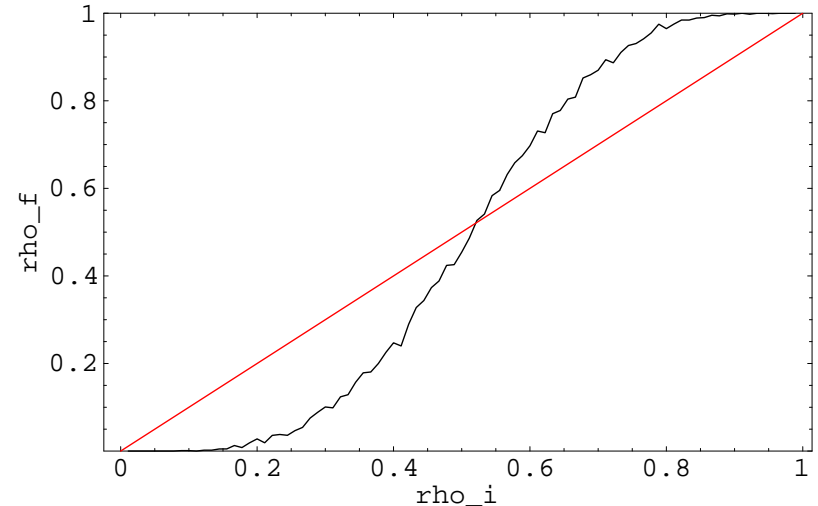
EXTREME BEVERAGE 00.1 COCKTAIL TIMES.COM

Highly Ordered Networks

- Add persons with high alcoholism tendency and observe the behavior of the network.
- When cave is full, add to the next cave.



Sequential seeding

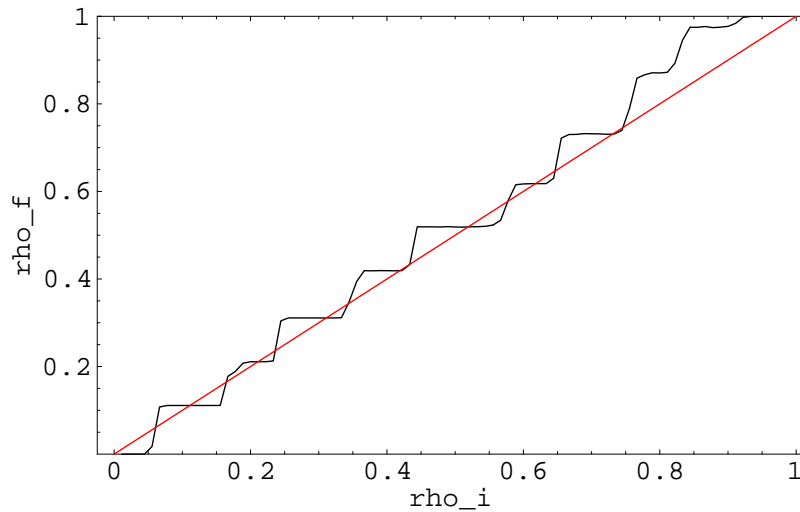


Random seeding

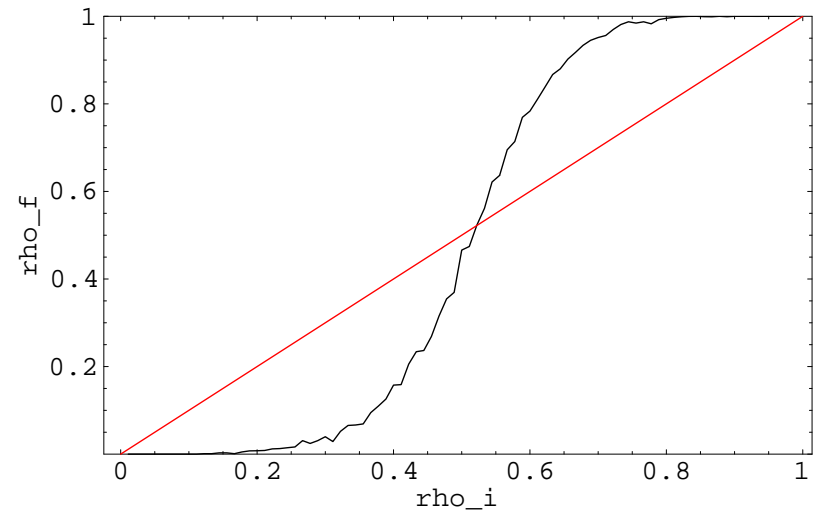
In a highly ordered network with a highly ordered stocking scheme, the response is linear.

20% Rewired Networks

Random links have been rewired to decrease the graph's diameter.



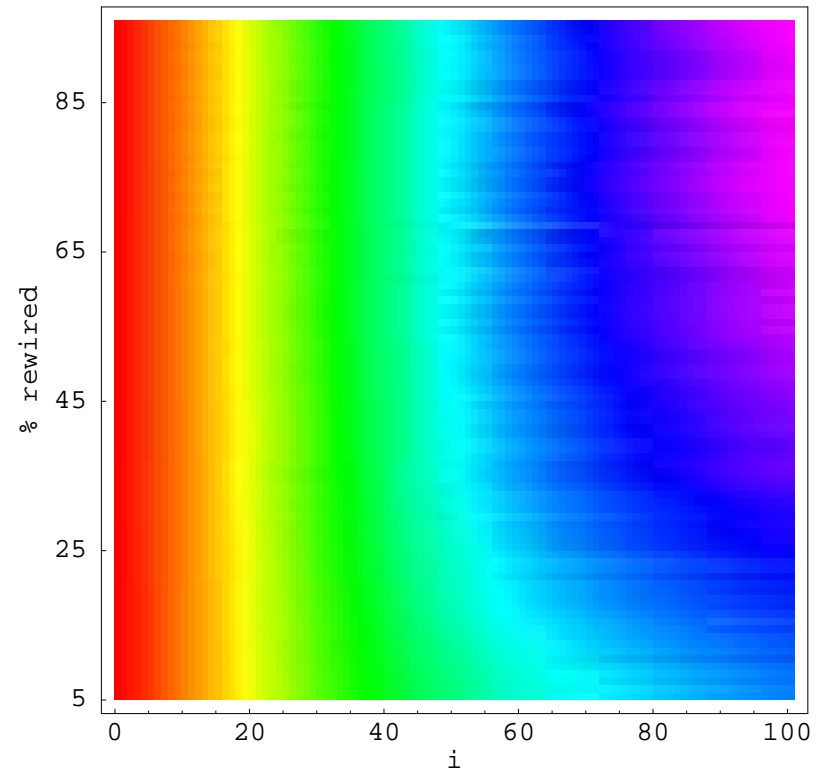
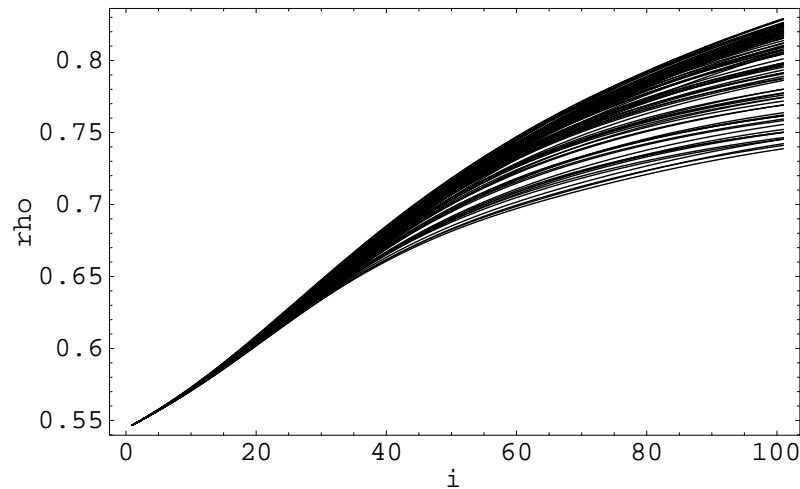
Sequential seeding



Random seeding

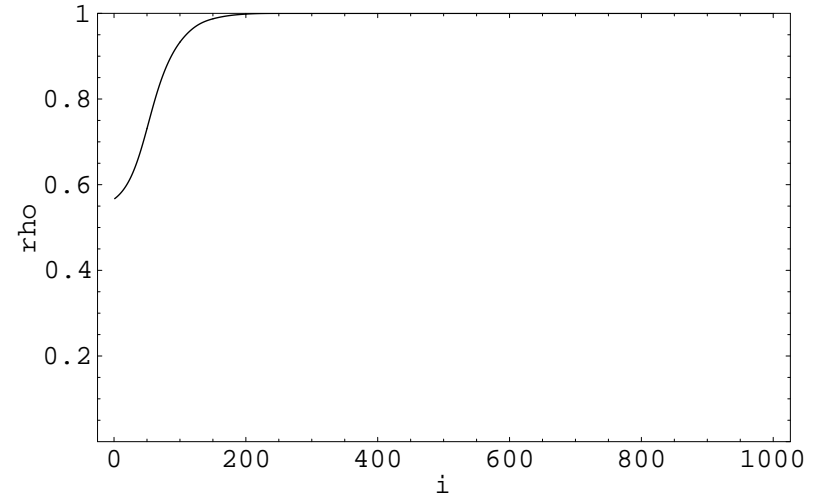
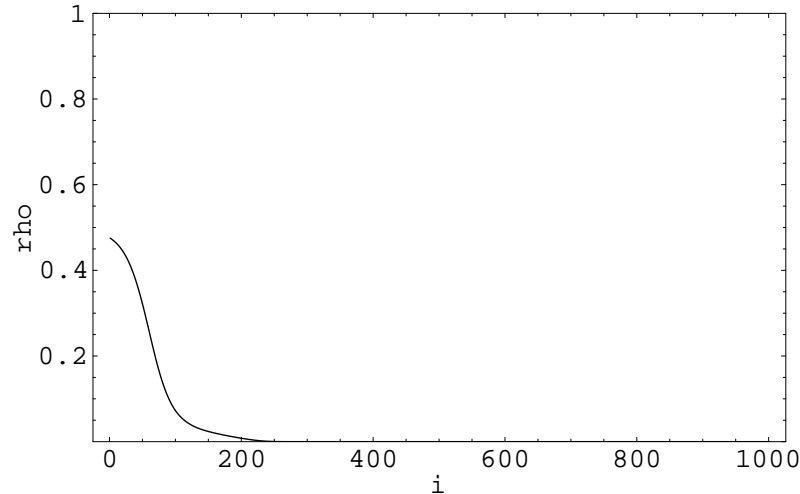
Even in a “small world” highly ordered stocking yields a nearly linear network response.

Increased Rewiring



Increasing the proportion of randomly linked vertices tends to shorten the time taken to reach steady-state.

Tipping Behavior

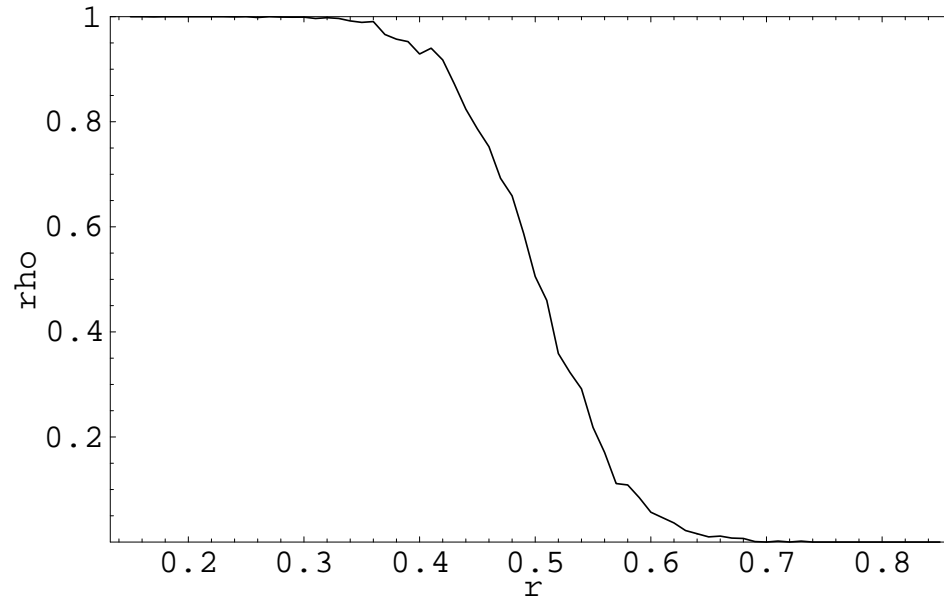


If all resiliencies equal the mean alcoholism level of neighbors, a(n) decrease (increase) in a single individual's alcoholism level leads to a cascading decrease (increase) in societal alcoholism.

Treatment Options

- Raise mean network resiliency
- Raise resiliency of vertices when they reach a certain threshold
- Lower the alcoholism level of top drinkers
- Rewire links of top drinkers

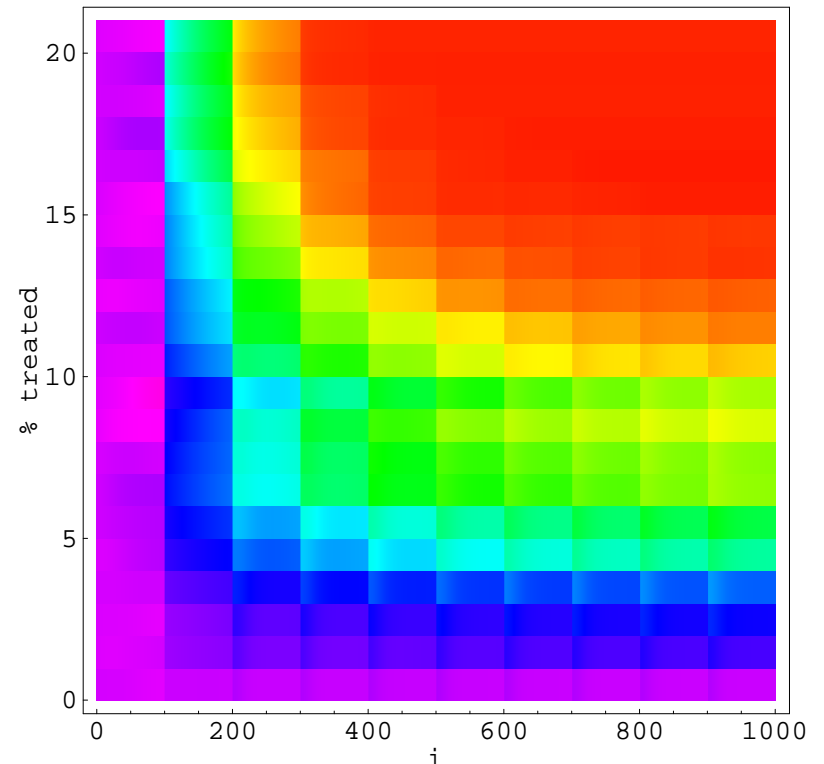
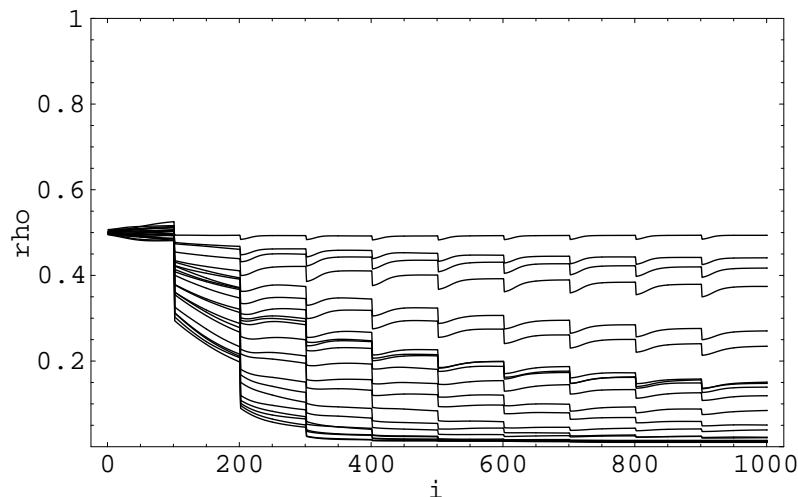
Raising Mean Resiliency



As mean resiliency increases from 0.15 to 0.85 the asymptotic level of alcoholism drops to near zero. Standard deviation in resiliency was kept at $\sigma = 1/6$ and all resiliencies were adjusted to lie in $[0, 1]$.

Withhold Alcohol

Periodically enforce a lower level of alcohol consumption for the heaviest $n\%$ of drinkers.

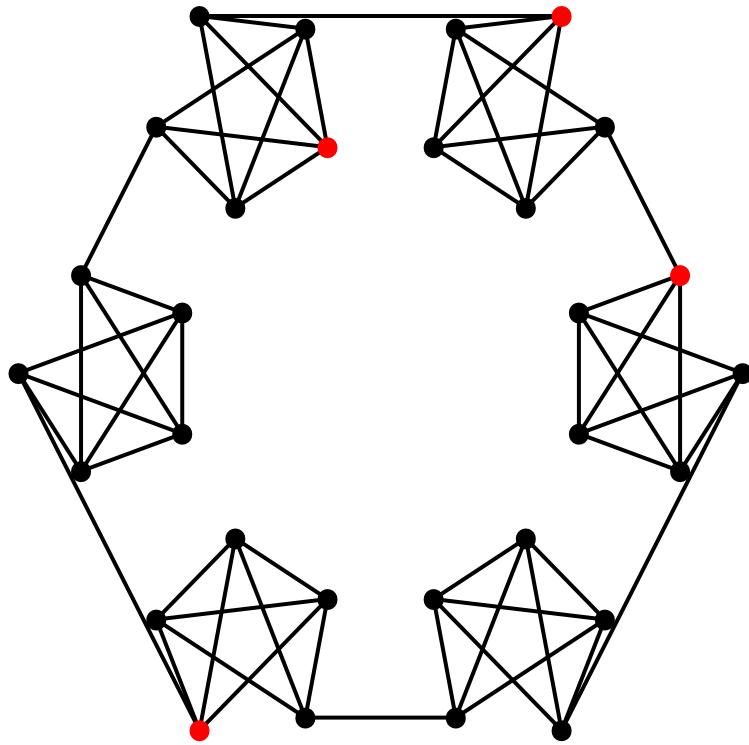


After a period of sobriety, the formerly heavy drinkers are allowed to resume independent behavior.

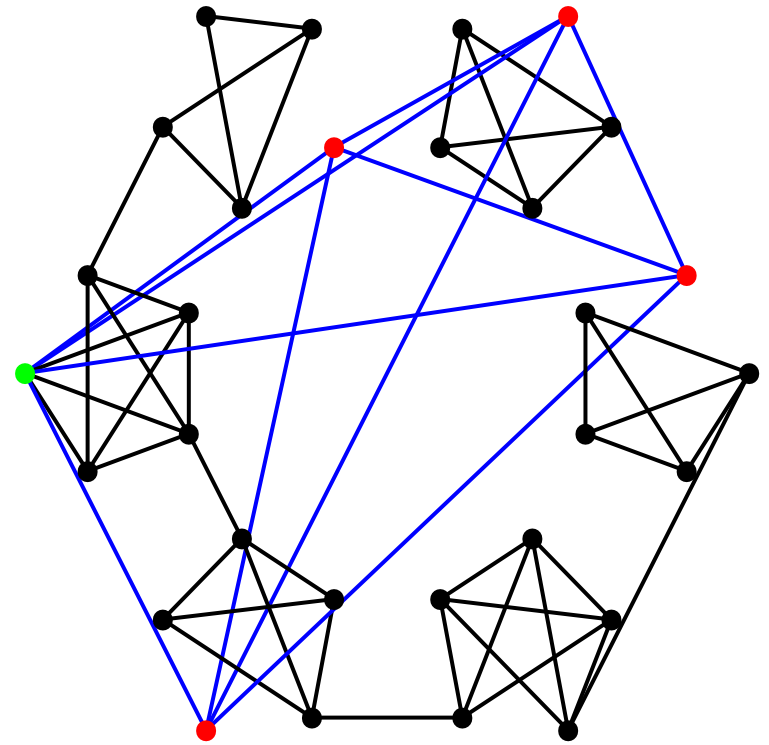
Rehabilitation I

- Periodically cluster the heaviest $n\%$ of drinkers.
- Link to each other and one other randomly chosen vertex.
- Sever all other links to the heaviest $n\%$ of drinkers.
- Enforce a lower level of alcohol consumption within the cluster.
- After the rehabilitation period ends, restore the original links.

Rehabilitation II

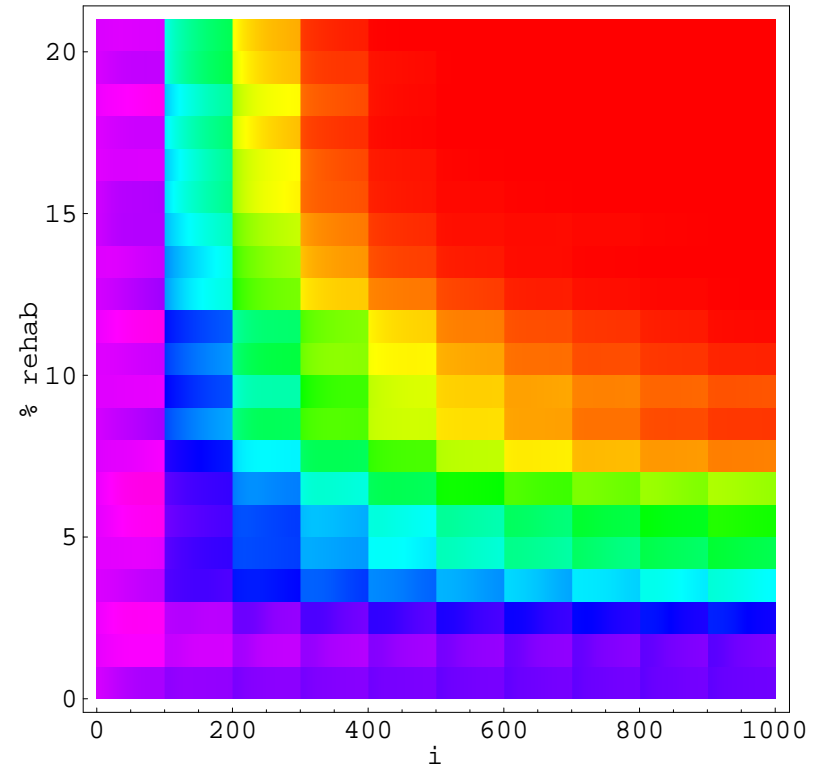
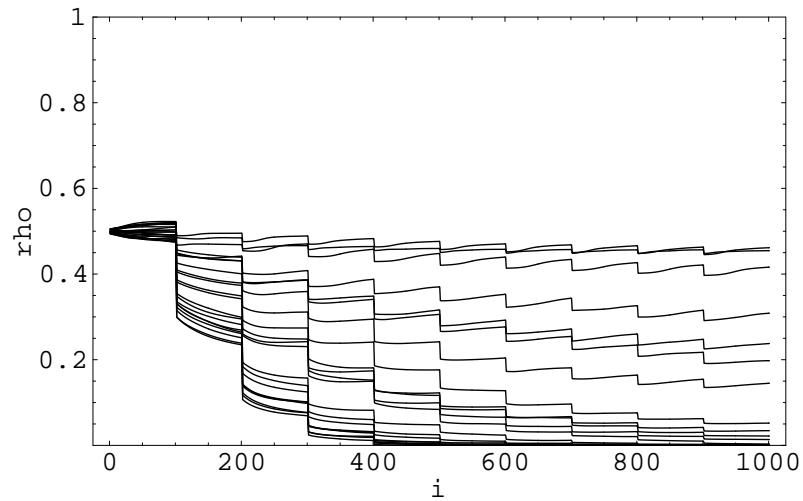


Before rehab



During rehab

Rehabilitation III



New group placed into rehab every 100 time steps.

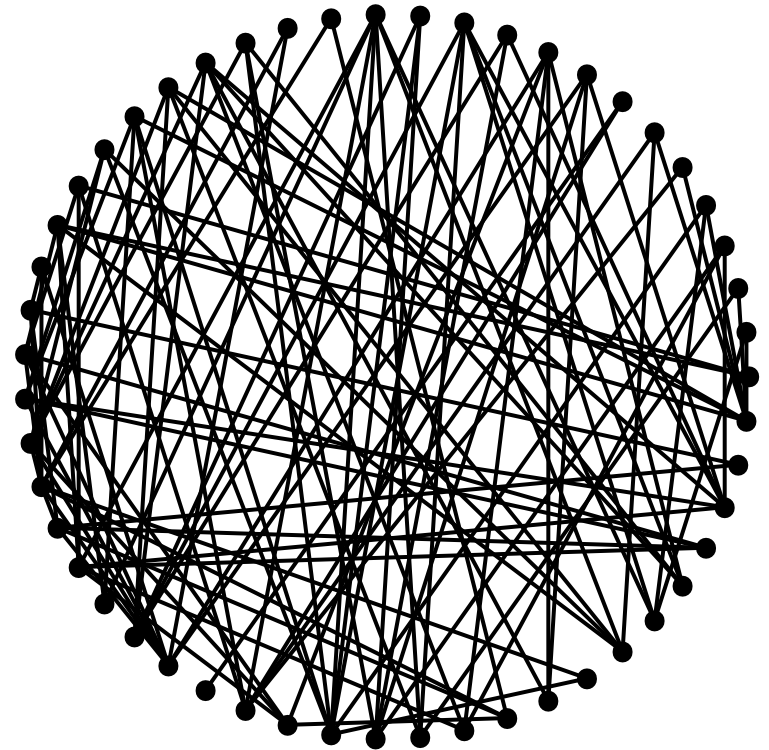
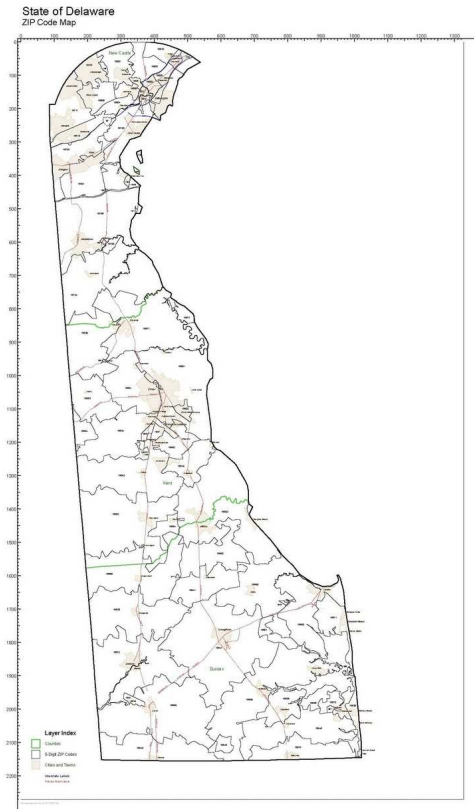
Future Work

- Explore the effects of other treatment policies.
- Develop better understanding of the behavior of the model as a discrete dynamical system.
- Scale model to realistic size community (city, county, state).
- Incorporate survey data from state of Delaware.
- Complete manuscript and submit to *Journal of Studies on Alcohol*.

Delaware Data

- Organized by zipcode
- Adjacency relationships for zipcodes included
- Includes gender, racial, marital, educational, age, income makeup of zipcodes
- Includes drinking behavior of respondents
 - Drinks in lifetime, previous year, number of drinks
 - Treatment for alcohol disorders in lifetime, past year
 - Alcohol dependence in lifetime, past year

Delaware Network



Zip code boundaries for Delaware and the induced network of zip codes.

Vodka Shot

For those people who don't like complicated recipes for drinks or don't like mixers ...

- **Ingredients:**

- 1 shot of vodka
- Hold glass containing shot at chest level.
- Say “na zdrowie”.
- Swallow entire contents in one motion.
- Repeat at least five times.