Ideas for Student Projects

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Home Equity Derivative

The world's economies are under stress triggered by falling home values in the USA.

- Some economists have suggested the creation of a "home equity derivative", a type of insurance which home owners could use to reduce the risk of losing wealth if the value of their home should decrease.
- How should the "home equity derivative" be priced?
- How should the risk of collapse in housing prices be hedged so that the seller can meet its obligation to homeowners?
- What sectors of the economy could the housing market be hedged against?

Non-lognormal Distributions of Changes in Security Prices

- If $\{S_n\}_{n=0}^{\infty}$ is a sequences of security prices it is generally assumed that $\ln(S_{n+1}/S_n)$ is a normally distributed random variable.
- On 10/19/1987 the S&P500 index fell 20.5%. Similar size changes occur about every 25 years based on historical data.
- The normal distribution would predict that changes of that size occur only every 2×10^{76} years.
- If the lognormal assumption is replaced what effect does this have on derivative prices?

Asian Option Pricing (1 of 2)

The Black-Scholes PDE governs the pricing of European options.

- Right to buy security currently values at $S(0)$ at time T,
- for price K,
- when risk-free interest rate is r, and
- the volatility in the stock price is σ^2.

$$C(S,t) = S\phi(w) - Ke^{-r(T-t)}\phi(w - \sigma\sqrt{T-t})$$

Asian Option Pricing (2 of 2)

An Asian option has a payoff which depends on the average price of the underlying stock. The option can be exercised at expiry (European), at specified times up to expiry (Bermudan), or any time prior to expiry (American).

- Become acquainted with finite difference PDE solution techniques.
- Gain an understanding of moving boundary PDEs.
- Develop a numerical method for pricing Asian options.
- Validate the numerical method against published prices.
- Develop a “nice” interface between the solver and *Mathematica*.

Stochastic Differential Equation Solver (1 of 2)

- Solutions to ODEs can exhibit intricate behavior (chaos, bifurcations, *etc.*) but contain only deterministic expressions.
- Stochastic differential equations (SDEs) contain deterministic and random expressions.

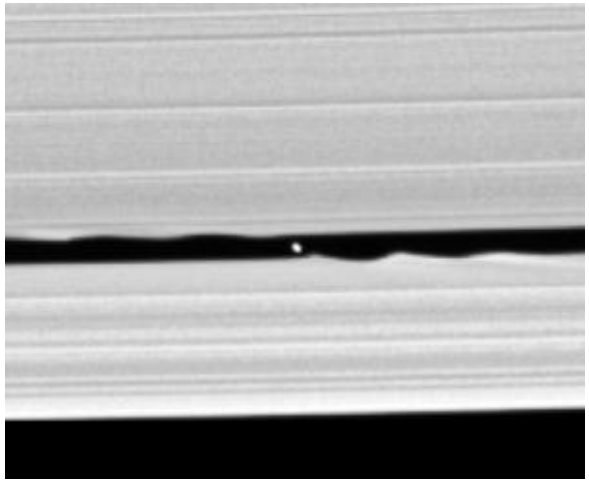
$$dX_t = a(X_t, t)\,dt + b(X_t, t)dW_t$$

- Solutions to SDEs are random variables.

Stochastic Differential Equation Solver (2 of 2)

- Build background in the theory of SDEs.
- Survey reduction and change of variable techniques for solving SDEs.
- Extend the analytical and/or numerical ODE solvers of *Mathematica* to handle SDEs.

Structure of Saturn's Rings



Many intricate structures can be seen in high resolution images of Saturn's rings.

Planar Three-Body Problem

- Develop a celestial mechanics model of Saturn, its moon Prometheus, and the dust in its rings.
- Analyze the dynamics of the particles in the rings.
- Create large-scale numerical simulation of the ring dynamics on a cluster of computers
- Attempt to re-create some of the features seen in the rings.

Temporal Synchrony (1 of 2)

- Some insect species are able to time their emergence from the larval stage so that a very large number of individuals appear at the same time. Pine bark beetles and cicadas do this.
- It is believed that fitness is enhanced by emerging en masse.
  - Larger numbers of insects may be able to overcome the defenses of vegetation that are resistant to small numbers of insects.
  - Having a large number of potential mates improves the chances of leaving offspring in the next generation.
  - There may also be safety in numbers from predators when a large population of identical insects emerges simultaneously.
- Biologists have a theory that this synchronized emergence is related to environmental temperature. When yearly average temperature deviates from normal, the yearly cycle of emergence can be broken.

Temporal Synchrony (2 of 2)

Project Description:

- The student will develop and analyze a mathematical model of an insect population which exhibits synchronized emergence.
- The student will study the mathematical models of insect populations and circle maps.
- The dynamical concept of the rotation number of a circle map may aid in the development and the analysis of the model.

Roundoff Errors and Chaos (1 of 2)

- Dynamical systems generated by difference equations can exhibit a range of dynamical behaviors, from fixed points to periodic orbits to strange attractors to chaos.
- Part of the definition of chaos is the idea known as sensitive dependence to initial conditions. If a chaotic system is given two arbitrarily close but different initial conditions then the two orbits will separate by an arbitrarily large distance.
- Due to roundoff errors in numerical calculations, dynamics may appear non-chaotic while if calculated exactly produce a chaotic orbit.

Roundoff Errors and Chaos (2 of 2)

Project Description:

- The student will study the behavior of difference equations such as the logistic equation.
- In particular the investigation will address the issue of the relationship between sensitive dependence to initial conditions, numerical precision, and coalescence (the behavior that results when two orbits which have different initial conditions have the same asymptotic orbit).