Hedging

An Undergraduate Introduction to Financial Mathematics

J. Robert Buchanan

2010
**Definition**

**Hedging** is the practice of making a portfolio of investments less sensitive to changes in market variables.

There are various **hedging strategies**.

For example, **Delta hedging** attempts to keep the $\Delta$ of a portfolio nearly 0, so that the value of the portfolio is insensitive to changes in the price of a security.
A bank sells a call option on a stock to an investor.
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If the investor chooses, they may purchase the stock at the strike price.
Responsibilities of the Seller of an Option

- A bank sells a call option on a stock to an investor.
- If the investor chooses, they may purchase the stock at the strike price.
  - If, at the strike time, the market price of the stock is below the strike price, the call option will not be exercised and the bank keeps the premium paid on the call by the investor.
A bank sells a call option on a stock to an investor.

If the investor chooses, they may purchase the stock at the strike price.

If, at the strike time, the market price of the stock is below the strike price, the call option will not be exercised and the bank keeps the premium paid on the call by the investor.

If the market price of the stock exceeds the strike price, the bank must ensure the investor can purchase the stock at the strike price.
A bank sells a call option on a stock to an investor.

If the investor chooses, they may purchase the stock at the strike price.

- If, at the strike time, the market price of the stock is below the strike price, the call option will not be exercised and the bank keeps the premium paid on the call by the investor.
- If the market price of the stock exceeds the strike price, the bank must ensure the investor can purchase the stock at the strike price.

The bank must sell the stock for the strike price to the investor.
Example

A bank sells $10^6$ European calls on a stock where $S(0) = $50, $K = $52, $r = 2.5\%$, $T = 1/3$, and $\sigma = 22.5\%$. According to the Black-Scholes option pricing formula, $C = $1.91965.
Example

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Suppose the bank purchases the $10^6$ shares of the security immediately (a **covered** position). At $T$ the net revenue satisfies:

$$1919650 + \left( \min\{52, S\} e^{-0.025/3 - 50} \right) \cdot 10^6.$$
If $S \approx 48.4827$ the net revenue is zero.
If $S = 46$ the loss to the bank is approximately $2.4M$.
If $S = 52$ the profit of the bank is nearly $3.5M$. 
Suppose the bank purchases the $10^6$ shares of the security at the strike time (called a **naked position**) and immediately sells them to the owner of the option. The net revenue satisfies:

$$1919650 + \min\{0, 52 - S\} e^{-0.025/3} \cdot 10^6.$$
Suppose the bank purchases the $10^6$ shares of the security at the strike time (called a \textbf{naked position}) and immediately sells them to the owner of the option. The net revenue satisfies:

$$1919650 + \min\{0, 52 - S\} e^{-0.025/3} \cdot 10^6.$$ 

This assumes the call option will be exercised.
Net revenue is zero when $S \approx $53.9357.

If $S = $56 the bank’s net loss would be approximately $2.0M.
Recall: If the value of a solution to the Black-Scholes PDE is $F$ then $\Delta = \frac{\partial F}{\partial S}$ where $S$ is the value of some security underlying $F$.

- If $F$ is an option then for every unit change in the value of the underlying security, the value of the option changes by $\Delta$.
- A portfolio consisting of securities and options is called **Delta-neutral** if for every option sold, $\Delta$ units of the security are bought.
Example of Delta Hedging

Example

Suppose $S = \$100$, $r = 4\%$, $\sigma = 23\%$, $K = \$105$, and $T = 1/4$. Under these conditions $w = -0.279806$, the value of a European call option is $C = 2.96155$ and Delta for the option is

$$\Delta = \frac{\partial C}{\partial S} = 0.389813.$$

Thus if a firm sold an investor European call options on ten thousand shares of the security, the firm would receive $29,615.50 and purchase $389,813 = (10000)(0.389813)(100)$ worth of the security.
Rebalancing a Portfolio

If, after setting up the hedge, a firm does nothing else until expiry, this is called a “hedge and forget” scheme.

The price of the security will (probably) change during the life of the option, so the firm may choose to make periodic adjustments to the number shares of the security it holds.
Rebalancing a Portfolio

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The price of the security will (probably) change during the life of the option, so the firm may choose to make periodic adjustments to the number shares of the security it holds.

This scheme is known as **rebalancing** the portfolio.
Assume the value of the security follows the random walk shown below.

The European call option will be exercised since \( S(T) > K \).
Suppose that $S(1/52) = \$98.79$.

Re-compute $\Delta = 0.339811$.

Sell $10000(0.389813 - 0.339811)$ shares of stock for $98.79 per share.

$$10000(0.389813 - 0.339811)(98.79) \approx 49,396.9758$$

Bank adjusts its stock holdings so that it now owns 3398.11 shares with a total value of $335,699.

Bank pays interest on the money borrowed to purchase the stock.

$$(389,813)(e^{0.04/52} - 1) = \$299.9717.$$ 

Cumulative costs to the bank at $t = 1/52$ are

$$389,813 + 299.9717 - 49,396.9758 \approx \$340,716.$$
## Weekly Rebalancing

<table>
<thead>
<tr>
<th>Week</th>
<th>S</th>
<th>Shares Held</th>
<th>Share Cost</th>
<th>Interest Cost</th>
<th>Cumulative Cost</th>
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<tbody>
<tr>
<td>0</td>
<td>100.00</td>
<td>0.389813</td>
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<td>10000.00</td>
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</table>
At expiry the net proceeds to the bank are

$$1,050,000 + 29,615.50e^{0.04(13/52)} - 1,069,458 = $10,455.10.$$
Second Extended Example

Suppose the call option expires without being exercised.

\[ S(T) < K \]
Suppose that $S(1/52) = $101.71.

Re-compute $\Delta = 0.440643$.

Buy $10000(0.440643 - 0.389813)$ shares of stock for $101.71$ per share.

$$10000(0.440643 - 0.389813)(101.71) \approx 57,191.533$$

Bank adjusts its stock holdings so that it now owns 4406.43 shares with a total value of $448,179$.

Bank pays interest on the money borrowed to purchase the stock.

$$(389,813)(e^{0.04/52} - 1) = $299.9717.$$ 

Cumulative costs to the bank at $t = 1/52$ are 

$$389,813 + 299.9717 + 57,191.533 \approx $441,813.$$
### Weekly Rebalancing

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<tr>
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<td>0.000000</td>
<td>0.000000</td>
<td>0.00000</td>
<td>29,667.7</td>
</tr>
</tbody>
</table>
At expiry the net proceeds to the bank are

\[29,615.50e^{0.04(13/52)} - 29,667.7 = 245.44.\]
Example

We have already seen that the values of European call and put options satisfy the Black-Scholes PDE.

\[ rF = Ft + \frac{1}{2} \sigma^2 S^2 F_{SS} + rSF_S \]

Show that the following are also solutions.

1. \( F(S, t) = S \)
2. \( F(S, t) = Ae^{rt} \)
Example

We have already seen that the values of European call and put options satisfy the Black-Scholes PDE.

\[ rF = F_t + \frac{1}{2} \sigma^2 S^2 F_{SS} + rSF_S \]

Show that the following are also solutions.

1. \( F(S, t) = S \)
2. \( F(S, t) = Ae^{rt} \)

Hence, the stock itself and cash are both solutions to the Black-Scholes PDE.
A portfolio consists of a short position in a European call option and a long position in the security (Delta hedged). Thus the net value $P$ of the portfolio is

$$P = C - \Delta S = C - \left. \frac{\partial C}{\partial S} \right|_{S_0} S.$$ 

$P$ satisfies the Black-Scholes equation since $C$ and $S$ separately solve it. Thus Delta for the portfolio is

$$\frac{\partial P}{\partial S} = \frac{\partial C}{\partial S} - \left. \frac{\partial C}{\partial S} \right|_{S_0}.$$ 

$$\frac{\partial P}{\partial S} \approx 0 \text{ when } S \approx S(0).$$
Taylor Series for $\mathcal{P}$

$$
\begin{align*}
\mathcal{P} &= \mathcal{P}_0 + \frac{\partial \mathcal{P}}{\partial t} (t - t_0) + \frac{\partial \mathcal{P}}{\partial S} (S - S_0) + \frac{\partial^2 \mathcal{P}}{\partial S^2} \frac{(S - S_0)^2}{2} + \cdots \\
\delta \mathcal{P} &= \Theta \delta t + \Delta \delta S + \frac{1}{2} \Gamma (\delta S)^2 + \cdots \\
\delta \mathcal{P} &\approx \Theta \delta t + \frac{1}{2} \Gamma (\delta S)^2
\end{align*}
$$

- $\Theta$ is not stochastic and thus must be retained.
- What about $\Gamma$?
Recall: \[ \Gamma = \frac{\partial^2 F}{\partial S^2} \]

- Since \( \frac{\partial^2}{\partial S^2}(S) = 0 \) a portfolio cannot be made gamma neutral if it contains only an option and its underlying security.
- Portfolio must include an additional component which depends non-linearly on \( S \).
- Portfolio can include two (or more) different types of option dependent on the same security.
Suppose a portfolio contains options with two different strike times written on the same stock.

A firm may sell European call options with a strike time three months and buy European call options on the same stock with a strike time of six months.

Let the number of the early option sold be $w_e$ and the number of the later option be $w_l$.

The Gamma of the portfolio would be

$$\Gamma_P = w_e \Gamma_e - w_l \Gamma_l,$$

where $\Gamma_e$ and $\Gamma_l$ denote the Gammas of the earlier and later options respectively.
Choose \( w_e \) and \( w_I \) so that \( \Gamma_P = 0 \).

Introduce the security so as to make the portfolio Delta neutral.

**Question:** Why does changing the number of shares of the security in the portfolio affect \( \Delta \) but not \( \Gamma \)?

\[ \delta P \approx \Theta \delta t. \]
Example (3 of 5)

- Suppose $S = $100, $\sigma = 0.22$, and $r = 2.5\%$.
- An investment firm sells a European call option on this stock with $T_3 = 1/4$ and $K = $102.
- The firm buys European call options on the same stock with the same strike price but with $T_6 = 1/2$.
- Gamma of the 3-month option is $\Gamma_3 = 0.03618$ and Gamma of the 6-month option is $\Gamma_6 = 0.02563$.
- The portfolio is Gamma neutral in the first quadrant of $w_3 w_6$-space where the equation
  
  \[ 0.03618 w_3 - 0.02563 w_6 = 0 \]
  
  is satisfied.
Suppose $w_3 = 100000$ of the three-month option were sold.

Portfolio is Gamma neutral if $w_6 = 141163$ six-month options are purchased.

Before including the underlying stock in the portfolio, the Delta of the portfolio is

$$w_3 \Delta_3 - w_6 \Delta_6 = (100000)(0.4728) - (141163)(0.5123) = -25038.$$ 

Portfolio can be made Delta neutral if 25,038 shares of the underlying stock are sold short.
Over a wide range of values for the underlying stock, the value of the portfolio remains nearly constant.
Rho and Vega can be used to hedge portfolios against changes in the interest rate and volatility respectively.

We have assumed that the necessary options and securities could be bought or sold so as to form the desired hedge.

If this is not true then a firm or investor may have to substitute a different, but related security or other financial instrument in order to set up the hedge.
Definition

A trading strategy involving two or more options of the same type on the same stock is called a spread.
Spreads

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Definition
A long call position with a strike price of $K_1$ and a short call position with a strike price of $K_2 > K_1$ is called a bull spread.

Remark: We are assuming the underlying stock is the same and the expiry of the two calls is the same.
Bull Spread (1 of 2)

- Since $K_2 > K_1$ then $C_2 < C_1$.
- Initial outlay of capital in amount $C_1 - C_2 > 0$.
- Payoff of long call $(S(T) - K_1)^+.$
- Payoff of short call $- (S(T) - K_2)^+$. 
Since $K_2 > K_1$ then $C_2 < C_1$.

Initial outlay of capital in amount $C_1 - C_2 > 0$.

Payoff of long call $(S(T) - K_1)^+$.

Payoff of short call $-(S(T) - K_2)^+$.

<table>
<thead>
<tr>
<th>$S(T)$</th>
<th>Long Call Payoff</th>
<th>Short Call Payoff</th>
<th>Total Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S(T) \leq K_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$K_1 &lt; S(T) &lt; K_2$</td>
<td>$S(T) - K_1$</td>
<td>0</td>
<td>$S(T) - K_1$</td>
</tr>
<tr>
<td>$K_2 \leq S(T)$</td>
<td>$S(T) - K_1$</td>
<td>$K_2 - S(T)$</td>
<td>$K_2 - K_1$</td>
</tr>
</tbody>
</table>
Bull Spread (2 of 2)

Payoff/Profit

\[ S(T) \]

\[ C_2 - C_1 \]

\[ K_1 \]

\[ K_2 \]
Example: Bull Spread

Example

Suppose we create a bull spread purchasing a call option with strike price $115 and selling a call option with strike price $130. Suppose that $C(115) = 5$ and $C(130) = 3$. Find the payoff and net profit if at expiry

- $S = 110$
- $S = 115$
- $S = 125$
- $S = 130$
- $S = 135$
Suppose an investor has a long position in a put with strike price $K_1$ and a short position in a put with strike price $K_2 > K_1$.

<table>
<thead>
<tr>
<th>$S(T)$</th>
<th>Long Put Payoff</th>
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</tr>
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<tbody>
<tr>
<td>$S(T) \leq K_1$</td>
<td>$K_1 - S(T)$</td>
<td>$S(T) - K_2$</td>
<td>$K_1 - K_2$</td>
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<tr>
<td>$K_1 &lt; S(T) &lt; K_2$</td>
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<td>$S(T) - K_2$</td>
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<tr>
<td>$K_2 \leq S(T)$</td>
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<td>$0$</td>
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</table>
Bull Spread with Puts (2 of 2)

Payoff/Profit

$P_2 - P_1$

$K_1$ $K_2$ $S(T)$

J. Robert Buchanan

Hedging
**Example**

Suppose we create a bull spread purchasing a put option with strike price $95 and selling a put option with strike price $105. Suppose that $P(95) = 5$ and $P(105) = 8$. Find the payoff and net profit if at expiry:

- $S = 90$
- $S = 95$
- $S = 100$
- $S = 105$
- $S = 110$
Bear Spreads

Definition

A short call position with a strike price of $K_1$ and a long call position with a strike price of $K_2 > K_1$ is called a bear spread.

Remarks:

- We are assuming the underlying stock is the same and the expiry of the two calls is the same.
- The positions in the bear spread are opposite those of the bull spread.
Since $K_2 > K_1$ then $C_2 < C_1$.

Initial income of capital in amount $C_1 - C_2 > 0$.

Payoff of long call $(S(T) - K_2)^+$. 

Payoff of short call $-(S(T) - K_1)^+$. 
Since $K_2 > K_1$ then $C_2 < C_1$.

Initial income of capital in amount $C_1 - C_2 > 0$.

Payoff of long call $(S(T) - K_2)^+.$

Payoff of short call $-(S(T) - K_1)^+.$

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<td>0</td>
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</table>
Bear Spread (2 of 2)

Payoff/Profit

$C_1 - C_2$

$K_1$ $K_2$

$S(T)$
Example: Bear Spread

Example

Suppose we create a bear spread purchasing a call option with strike price $150 and selling a call option with strike price $125. Suppose that \( C(150) = 5 \) and \( C(125) = 10 \). Find the payoff and net profit if at expiry

- \( S = 120 \)
- \( S = 125 \)
- \( S = 150 \)
- \( S = 150 \)
- \( S = 160 \)
Suppose an investor has a short position in a put with strike price $K_1$ and a long position in a put with strike price $K_2 > K_1$.

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<td>0</td>
<td>0</td>
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</tbody>
</table>
Payoff/Profit

$P_2 - P_1$

$K_1$

$K_2$

$S(T)$

$P_2 - P_1$

$K_1$

$K_2$

$S(T)$
Example: Bear Spread

Example

Suppose we create a bear spread selling a put option with strike price $50 and purchasing a put option with strike price $60. Suppose that $P(50) = 3$ and $P(60) = 5$. Find the payoff and net profit if at expiry

- $S = 45$
- $S = 50$
- $S = 55$
- $S = 60$
- $S = 65$
**Definition**

A long call position with a strike price of $K_1$, a long call position with a strike price of $K_3 > K_1$, and a short position in two calls with strike price $K_2 = (K_1 + K_3)/2$ is called a **butterfly spread**.

**Remark:** We are assuming the underlying stock is the same and the expiry of the three calls is the same.
Butterfly Spread

- Note that $K_3 > K_2 > K_1$.
- Payoffs of long calls are $(S(T) - K_1)^+$ and $(S(T) - K_3)^+$.
- Payoff of two short calls $-2(S(T) - K_2)^+$. 
Note that $K_3 > K_2 > K_1$.

Payoffs of long calls are $(S(T) - K_1)^+$ and $(S(T) - K_3)^+$.

Payoff of two short calls $-2(S(T) - K_2)^+$. 

<table>
<thead>
<tr>
<th>$S(T)$</th>
<th>Payoff 1st Long Call</th>
<th>Payoff 2nd Long Call</th>
<th>Payoff Short Calls</th>
<th>Payoff Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S(T) \leq K_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$K_1 &lt; S(T) &lt; K_2$</td>
<td>$S(T) - K_1$</td>
<td>0</td>
<td>0</td>
<td>$S(T) - K_1$</td>
</tr>
<tr>
<td>$K_2 \leq S(T) &lt; K_3$</td>
<td>$S(T) - K_1$</td>
<td>0</td>
<td>$-2(S(T) - K_2)$</td>
<td>$K_3 - S(T)$</td>
</tr>
<tr>
<td>$K_3 \leq S(T)$</td>
<td>$S(T) - K_1$</td>
<td>$S(T) - K_3$</td>
<td>$-2(S(T) - K_2)$</td>
<td>0</td>
</tr>
</tbody>
</table>
Butterfly Spread with Calls

\[ \text{Payoff/Profit} \]

\[ C_1 - 2C_2 + C_3 \]

\[ S(T) \]
Example: Butterfly Spread

Example

Suppose we create a butterfly spread purchasing a call options with strike prices $150 and $200 selling 2 call options with strike price $175. Suppose that \( C(150) = 60 \), \( C(175) = 35 \), and \( C(200) = 10 \). Find the payoff and net profit if at expiry

- \( S = 100 \)
- \( S = 150 \)
- \( S = 175 \)
- \( S = 200 \)
- \( S = 250 \)
Straddles

Definition

Simultaneous long position in a call and a put is called a long straddle. Simultaneous short position in a call and a put is called a short straddle.

Remarks: We will assume
- the underlying stock is the same for both options,
- the strike prices are the same,
- the expiry dates are the same.
Suppose an investor has a long straddle with strike prices $K$.

<table>
<thead>
<tr>
<th>$S(T)$</th>
<th>Put Payoff</th>
<th>Call Payoff</th>
<th>Total Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S(T) \leq K$</td>
<td>$K - S(T)$</td>
<td>0</td>
<td>$K - S(T)$</td>
</tr>
<tr>
<td>$K &lt; S(T)$</td>
<td>0</td>
<td>$S(T) - K$</td>
<td>$S(T) - K$</td>
</tr>
</tbody>
</table>
Long Straddle (2 of 2)

Payoff/Profit

\[
\text{Payoff/Profit} = \begin{cases} 
K - (C + P) & \text{if } S(T) < K - (C + P) \\
-(C + P) & \text{if } K - (C + P) < S(T) < K + (C + P) \\
K + (C + P) & \text{if } S(T) > K + (C + P) 
\end{cases}
\]

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Example: Long Straddle

**Example**

Suppose we create a long straddle purchasing call and put options with strike prices of $150. Suppose that \( C(150) = 20 \) and \( P(150) = 10 \). Find the payoff and net profit if at expiry

- \( S = 100 \)
- \( S = 140 \)
- \( S = 150 \)
- \( S = 160 \)
- \( S = 200 \)
Strangles

Definition

A long strangle is a simultaneous long position in a call and a put. A short strangle is a simultaneous short position in a call and a put.

Remarks: We will assume
- the underlying stock is the same for both options,
- the expiry dates are the same,
- the strike prices may be different (this characteristic distinguishes a strangle from a straddle).
Suppose an investor has a long strangle with strike prices $K_P$ for the put and $K_C > K_P$ for the call.

<table>
<thead>
<tr>
<th>$S(T)$</th>
<th>Put Payoff</th>
<th>Call Payoff</th>
<th>Total Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S(T) \leq K_P$</td>
<td>$K_P - S(T)$</td>
<td>0</td>
<td>$K_P - S(T)$</td>
</tr>
<tr>
<td>$K_P &lt; S(T) &lt; K_C$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$K_C &lt; S(T)$</td>
<td>0</td>
<td>$S(T) - K_C$</td>
<td>$S(T) - K_C$</td>
</tr>
</tbody>
</table>

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Long Strangle (2 of 2)

Payoff/Profit

\[ K_P \]

\[ K_C \]

\[ C+P \]

\[ S(T) \]
Example: Long Strangle

Example
Suppose we create a long strangle purchasing a call option with a strike price of $150 and a put option with a strike price of $130. Suppose that $C(150) = 20$ and $P(130) = 10$. Find the payoff and net profit if at expiry

- $S = 100$
- $S = 140$
- $S = 150$
- $S = 160$
- $S = 200$