

## Errata

Catching all the errors of both statement and omission in a book is a collaborative effort between the author of the book and its readers. As an author I am grateful to the readers who have taken the time to make me aware of misstatements in *An Undergraduate Introduction to Financial Mathematics*, (World Scientific Publishing Company, Hackensack, NJ, USA, ISBN: 981-256-637-6 (2006)). With the help of the readers I hope to keep improving this book. On this page of errata I have listed all the corrections made to the text as May 21, 2008.

### Chapter 1

Thanks to Catherine Albright who pointed out the following errors.

Page 3, line -3: “sample” should be “same”

Page 11, line -10: “as as” should be “as an”

Page 12, line 16: “in negative” should read “is negative”

Thanks to Prof. M.M. Chawla who pointed out the following errors.

Example 1.4: the author neglected to mention that the retiree plans to draw on the retirement fund until age 95.

The displayed equation at the bottom of page 8 should read as follows:

$$\begin{aligned} x \sum_{i=1}^{480} \left(1 + \frac{0.10}{12}\right)^{-i} &= x \left(1 + \frac{0.10}{12}\right)^{-1} \frac{1 - \left(1 + \frac{0.10}{12}\right)^{-480}}{1 - \left(1 + \frac{0.10}{12}\right)^{-1}} \\ &\approx 117.765x. \end{aligned}$$

The displayed equation at the bottom of page 10 should read as follows:

$$\begin{aligned} x \sum_{i=1}^{480} \left(1 + \frac{0.13}{12}\right)^{-i} &= x \left(1 + \frac{0.13}{12}\right)^{-1} \frac{1 - \left(1 + \frac{0.13}{12}\right)^{-480}}{1 - \left(1 + \frac{0.13}{12}\right)^{-1}} \\ &\approx 91.784x. \end{aligned}$$

The second displayed equation on page 11 should be

$$1500 \left(1 + \frac{0.03}{12}\right)^{-840} \approx 184.17.$$

### Chapter 2

page 21, line 6: “we as” should read “we have as”

Thanks to Prof. M.M. Chawla who pointed out the following errors.

page 17, line -13: “or 2 or 12” should be “of 2 or 12”.

page 19, line 6: “contestants” should be “contestant’s”

page 21, line 8: “is is” should be “it is”

Thanks to student Catherine Albright who pointed out the following error.

page 211, line 1: the P (0 black) should be calculated as  $P(0 \text{ black}) = \frac{5}{20} \cdot \frac{4}{19} \cdot \frac{3}{18} = \frac{1}{114}$

### Chapter 3

Thanks to student Catherine Albright who pointed out the following error.

page 35, line -6: “that if consider” should be “that if we consider”

page 39, line -8: The set  $[1, \infty) \times [-1, 2]$  should be  $[1, \infty) \times [-1, 2]$ .

page 44: the first line of the second displayed equation should read

$$\frac{\sqrt{2\pi}e^{-n}n^{n+1/2} \left(\frac{1}{2}\right)^n}{\sqrt{2\pi}e^{-(n+m)/2} \left(\frac{1}{2}(n+m)\right)^{(n+m+1)/2} \sqrt{2\pi}e^{-(n-m)/2} \left(\frac{1}{2}(n-m)\right)^{(n-m+1)/2}}$$

Thanks to Prof. M.M. Chawla who pointed out the following errors.

page 39: the third displayed equation should read

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1.$$

page 46, line 1: “ $y = r \cos \theta$ ” should be “ $y = r \sin \theta$ ”.

page 47: the second displayed equation should read as follows.

$$\begin{aligned} \text{Var}(X) &= \int_{-\infty}^{\infty} \frac{x^2}{2\sqrt{k\pi t}} e^{-\frac{x^2}{4kt}} dx - (\mathbb{E}[X])^2 \\ &= \frac{1}{\sqrt{k\pi t}} \lim_{M \rightarrow \infty} \int_0^M x^2 e^{-\frac{x^2}{4kt}} dx \\ &= \frac{1}{\sqrt{k\pi t}} \lim_{M \rightarrow \infty} (-2kt) \left( M e^{-M^2/4kt} - \int_0^M e^{-x^2/4kt} dx \right) \\ &= \frac{2kt}{\sqrt{k\pi t}} \int_0^{\infty} e^{-x^2/4kt} dx \\ &= 2kt \cdot \frac{1}{2\sqrt{k\pi t}} \int_{-\infty}^{\infty} e^{-x^2/4kt} dx \\ &= 2kt \end{aligned}$$

page 55, line 5: the approximation “ $0.562^2 \approx 0.316$ ” should be “ $0.582^2 \approx 0.316$ ”

page 60: the third displayed equation should read

$$\begin{aligned}
 E[(X - K)^+] &= \frac{1}{\sqrt{2\pi}} \int_{(\ln K - \mu)/\sigma}^{\infty} (e^{\sigma z + \mu} - K)^2 e^{-z^2/2} dz \\
 &= \frac{e^{2(\mu + \sigma^2)}}{\sqrt{2\pi}} \int_{(\ln K - \mu)/\sigma}^{\infty} e^{-(z - 2\sigma)^2/2} dz \\
 &\quad - \frac{2Ke^{\mu + \sigma^2/2}}{\sqrt{2\pi}} \int_{(\ln K - \mu)/\sigma}^{\infty} e^{-(z - \sigma)^2/2} dz \\
 &\quad + \frac{K^2}{\sqrt{2\pi}} \int_{(\ln K - \mu)/\sigma}^{\infty} e^{-z^2/2} dz \\
 &= e^{2(\mu + \sigma^2)} \phi\left(\frac{\mu - \ln K}{\sigma} + 2\sigma\right) + K^2 \phi\left(\frac{\mu - \ln K}{\sigma}\right) \\
 &\quad - 2Ke^{\mu + \sigma^2/2} \phi\left(\frac{\mu - \ln K}{\sigma} + \sigma\right).
 \end{aligned}$$

Tyler Coats of the USNA pointed out the following error in the first displayed equation on page 47.

The expression  $\sqrt{\frac{k\pi}{t}}$  should be  $\sqrt{\frac{kt}{\pi}}$ .

#### Chapter 4

Thanks to Prof. M.M. Chawla who pointed out the following errors.

page 67, line -4: vector  $\mathbf{y}$  has  $m$  (not  $n$ ) components.

page 68, Example 4.1: every mention of  $y_2$  should be  $y_1$ .

page 68, line -3: “at least 3” should be “at least 6”

page 70, line -4: the equality should read as follows:

$$2 - 2(3) = -4 < 1$$

page 225, Exercise 1: the payoffs should read as follows

Winning Outcome	Payoff
A	$2x - y - z$
B	$-x + 3y - z$
C	$-x - y + z$

A wagering scheme with positive payoff regardless of outcome is  $x = -3$ ,  $y = -2.5$ , and  $z = -5$ .

page 230, line -1:  $\langle 1, 1 \rangle$  should be  $\langle 2, 2 \rangle$ .

page 231, line 1:  $\langle 1, 1 \rangle$  should be  $\langle 2, 2 \rangle$ .

Prof. Josef Dick pointed out the following errors:

page 73: the two displayed expressions in the proof form the end of a sentence and thus should be followed by periods.

page 230: vector  $\mathbf{b} = \langle 4, 6 \rangle$  and on line 6, “ $y_1 + 6y_2$ ” should be “ $4y_1 + 6y_2$ ”.

## Chapter 5

Prof. Josef Dick pointed out the following errors:

page 77, line -3: “to a single unit” should read “to be a single unit”

page 78, line -3: “with the probability” should be “with the same probability”

page 83, line -7: the un-numbered displayed equation should state

$$\begin{aligned} P(S(n) = A \wedge m_{n-1} > A | S(0) = i) = \\ P(S(n) = 0 \wedge m_{n-1} > 0 | S(0) = i - A). \end{aligned}$$

page 234: the displayed equation in the center of the page should end with a period.

page 235, exercise (10) and (11): displayed equations should end with a period.

Thanks to Prof. M.M. Chawla who pointed out the following errors.

page 87, line 10: “addition” should be “additional”

page 89, line 13: “method if called” should read “method is called”

page 90, line -10: “of the step” should read “of the steps”

page 100, line -11: “this a” should read “this is a”

page 235, exercise (12): the displayed equation should be

$$\begin{aligned} dY &= \left( \mu P \left[ \frac{1}{P} \right] + \frac{1}{2} (\sigma P)^2 \left[ \frac{-1}{P^2} \right] \right) dt + \sigma P \left[ \frac{1}{P} \right] dW(t) \\ &= \left( \mu - \frac{\sigma^2}{2} \right) dt + \sigma dW(t) \end{aligned}$$

## Chapter 6

Thanks to Prof. M.M. Chawla who pointed out the following errors.

page 104, line -5: “some these” should read “some of these”

page 108, line 11: “At time  $T$ ” should be “At time  $t = 0$ ”

page 108, line 18: “at time  $T$ ” should be “at time  $t = 0$ ”

page 108, line -3: “a time” should read “at time”

page 110, Eq. (6.12): the operation  $+\Delta$  in the fourth line should be  $-\Delta$

page 111, line -14: “in the the equation” should be “in the equation”

page 112, line -2: a backwards parabolic equation is one in which  $u_t$  and  $u_{xx}$  have the same algebraic sign when appearing on the same side of the equals sign in the PDE.

page 235, line -5: “in either the case” should read “in either case”

page 237, line -3: “at the investor” should be “and the investor”

page 238, line -3: along the boundary where  $S = 0$  the value of the put option is the present value of the strike price,

$$F(0, t) = Ke^{-r(T-t)}.$$

Thanks to Prof. Josef Dick pointed out the following errors:

page 107, lines  $-2$  and  $-1$ : should read “The stock will be worth \$200 after time  $T$  with probability  $p$  or will be worth \$50 with probability  $1-p$ .”

page 100, Eq. (6.11): there is an extraneous comma before  $dW(t)$ .

page 236, line  $-5$ : the displayed equation should state

$$(C - S(0))e^{rt^*} + \min\{S(t^*), K\}.$$

Other typographical errors include the following.

page 236, line 5: “the the security” should be “the security”

## Chapter 7

Thanks to Prof. M.M. Chawla who pointed out the following errors.

page 115, line -4: “numbers. and that The” should be “numbers and that the”

page 116, line -5: “real-values” should read “real-valued”

page 122, the second displayed set of equations should read as follows:

$$\begin{aligned}
 v(x, 0) &= (e^x - 1)^+ \\
 u(x, 0) &= e^{(k-1)x/2}(e^x - 1)^+ \\
 &= e^{(k-1)x/2} \begin{cases} e^x - 1 & \text{if } x > 0, \\ 0 & \text{if } x \leq 0. \end{cases} \\
 &= \begin{cases} e^{(k+1)x/2} - e^{(k-1)x/2} & \text{if } x > 0, \\ 0 & \text{if } x \leq 0. \end{cases} \\
 &= (e^{(k+1)x/2} - e^{(k-1)x/2})^+
 \end{aligned}$$

page 123, Eq. (7.28): the first line of the displayed equation should read

$$\mathcal{F}\{u_\tau\} = \mathcal{F}\{u_{xx}\}$$

page 128, Exercise (7): the displayed expression should be

$$\left( e^{(k+1)(x+\sqrt{2\tau}y)/2} - e^{(k-1)(x+\sqrt{2\tau}y)/2} \right)^+$$

page 243, Exercise (10): throughout the solution  $w$  should be  $w'$ .

page 244, Exercise (14): the solution is  $\sigma \approx 0.396436$

Thanks to Alexander van Haastrecht, a student at the Vrije Universiteit, Amsterdam for finding the following errors in the text.

page 119, Eq. (7.6): the expression  $(K - S)^+$  should be  $(S - K)^+$ .

page 121, Eqs. (7.20)–(7.23) the expression  $e^{\alpha x + \beta t}$  should be  $e^{\alpha x + \beta \tau}$ .

## Chapter 8

Thanks to Prof. M.M. Chawla who pointed out the following errors.

page 131, line 6: “will be use extensively” should read “will be used extensively”

page 131, line -14: “derivative who value” should be “derivative whose value”

page 131, line -11: “In the Chapter 9” should read “In Chapter 9”

page 137, line -1: “who current value” should be “whose current value”

page 140, line -15: “value underlying” should read “value of the underlying”

page 141, line -4: “security who current” should be “security whose current”

page 142, line 1: “security who” should be “security whose”

page 142, line 5: “security who” should be “security whose”

page 246: the first displayed equation should be as follows:

$$\begin{aligned}
\frac{\partial P}{\partial t} &= \frac{Se^{-w^2/2} - Ke^{-r(T-t)-(w-\sigma\sqrt{T-t})^2/2}}{2\sigma\sqrt{2\pi(T-t)}} \left( \frac{\ln(S/K)}{T-t} - r - \sigma^2/2 \right) \\
&\quad - Ke^{-r(T-t)} \left( r\phi(w - \sigma\sqrt{T-t}) + \frac{\sigma e^{-(w-\sigma\sqrt{T-t})^2/2}}{2\sqrt{2\pi(T-t)}} \right) \\
&\quad + Kre^{-r(T-t)} \\
&= \frac{Se^{-w^2/2} - Ke^{-r(T-t)-(w-\sigma\sqrt{T-t})^2/2}}{2\sigma\sqrt{2\pi(T-t)}} \left( \frac{\ln(S/K)}{T-t} - r - \sigma^2/2 \right) \\
&\quad + Kre^{-r(T-t)} \left( 1 - \phi(w - \sigma\sqrt{T-t}) \right) \\
&\quad - \frac{\sigma Ke^{-r(T-t)-(w-\sigma\sqrt{T-t})^2/2}}{2\sqrt{2\pi(T-t)}} \\
&= \frac{Se^{-w^2/2}}{2\sigma\sqrt{2\pi(T-t)}} \left( \frac{\ln(S/K)}{T-t} - r - \sigma^2/2 \right) \\
&\quad + Kre^{-r(T-t)} \phi(\sigma\sqrt{T-t} - w) \\
&\quad - \frac{Ke^{-r(T-t)-(w-\sigma\sqrt{T-t})^2/2}}{2\sqrt{2\pi(T-t)}} \left( \frac{\ln(S/K)}{\sigma(T-t)} - r/\sigma - \sigma/2 + \sigma \right) \\
&= \frac{Se^{-w^2/2}}{2\sigma\sqrt{2\pi(T-t)}} \left( \frac{\ln(S/K)}{T-t} - r - \sigma^2/2 \right) \\
&\quad + Kre^{-r(T-t)} \phi(\sigma\sqrt{T-t} - w) \\
&\quad - \frac{Ke^{-r(T-t)-(w-\sigma\sqrt{T-t})^2/2}}{2\sigma\sqrt{2\pi(T-t)}} \left( \frac{\ln(S/K)}{T-t} - r + \sigma^2/2 \right)
\end{aligned}$$

Thanks to Prof. Josef Dick who pointed out the following errors:

Page 134, lines 6 and 7: in the derivation of Delta for the European call these lines should be

$$\begin{aligned}
\frac{\partial C}{\partial S} &= \phi(w) + \frac{e^{-w^2/2}}{\sigma\sqrt{2\pi(T-t)}} \times \\
&\quad \left( 1 - \frac{Ke^{-r(T-t)}}{S} e^{-(\sigma^2(T-t)-2(r+\sigma^2/2)(T-t)-2\ln(K/S))/2} \right)
\end{aligned}$$

## Chapter 9

Thanks to Prof. Josef Dick who pointed out the following errors.

page 143, line 10: “when a one entity” should be “when one entity”

Thanks to Prof. M.M. Chawla who pointed out the following errors.

page 144, line -14: “profit nearly” should be “profit is nearly”

page 149, line -7: “Thus it reasonable” should read “Thus it is reasonable”

page 151, line -13: “a way as to make” should be “a way so as to make”

## Chapter 10

Thanks to Prof. M.M. Chawla who pointed out the following errors.

page 158, line -5: “holds all finite” should be “holds for all finite”

page 164: the first displayed equation should read as follows.

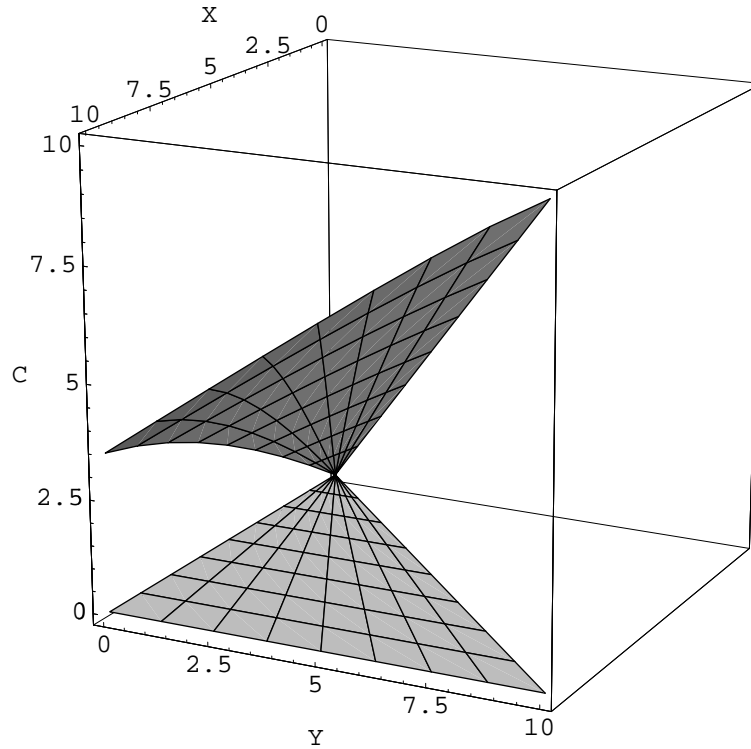
$$\begin{aligned} E [X(X - K)^+] &= \frac{e^{2(\mu+\sigma^2)}}{\sqrt{2\pi}} \int_{(\ln K - \mu)/\sigma}^{\infty} e^{-(z-2\sigma)^2/2} dz \\ &\quad - \frac{K e^{\mu+\sigma^2/2}}{\sqrt{2\pi}} \int_{(\ln K - \mu)/\sigma}^{\infty} e^{-(z-\sigma)^2/2} dz \\ &= e^{2(\mu+\sigma^2)} \phi \left( \frac{\mu - \ln K}{\sigma} + 2\sigma \right) \\ &\quad - K e^{\mu+\sigma^2/2} \phi \left( \frac{\mu - \ln K}{\sigma} + \sigma \right). \end{aligned}$$

page 168, line 7: “If  $f \mathcal{C}^2(a, b)$  then” should be “If  $f \in \mathcal{C}^2(a, b)$  then”

page 173: the first displayed equation is solved incorrectly. This will also change the paragraph following the equation and Fig. 10.4 on page 174. They should be as follows.

$$\begin{aligned} C - \frac{C^2}{25} &= \frac{1}{2} \left( X - \frac{X^2}{25} + Y - \frac{Y^2}{25} \right) \\ \left( C - \frac{25}{2} \right)^2 &= \frac{1}{2} \left[ \left( X - \frac{25}{2} \right)^2 + \left( Y - \frac{25}{2} \right)^2 \right] \\ C &= \frac{25}{2} - \frac{1}{\sqrt{2}} \sqrt{\left( X - \frac{25}{2} \right)^2 + \left( Y - \frac{25}{2} \right)^2} \end{aligned}$$

Again note that we choose the certainty equivalent to be the smallest value of  $C$  satisfying the equation. The design of investment A specifies that  $0 < Y < X < 10$ . Thus the certainty equivalent can be thought of as a surface plotted over the triangular region bounded by  $0 < X < 10$  with  $0 < Y < X$ . Figure 10.4 illustrates the certainty equivalent as a function of  $X$  and  $Y$ .



page 173, line -8: “the proportional they” should read as “the proportion they”

page 175, line 4: the curve is a function of  $p$  only

page 178, line 2: it should be made explicit that  $\sigma_1^2 + \sigma_2^2 - 2c$  is the covariance of  $R_1 - R_2$  with itself

page 179, line 4: “rates are return” should be “rates of return”

page 185, line 12: “and a the” should read “and the”

page 186, line 8: “ $\rho_{i,m} = 1$ ” should be “ $\rho_{i,M} = 1$ ”

page 186: the 3<sup>rd</sup> displayed equation should be

$$r = r_M - \frac{(r_i - r_M)(-\sigma_M \pm \sigma)}{\sigma_i + \sigma_M}.$$

page 193, line 8: “in a fair” should read “in which a fair”

page 193, line 15: the simple interest rate is omitted, it should be  $r = 11\%$ .

page 255, line -3: “which implied” should be “which implies”

page 258: the first displayed equation should be as follows:

$$\begin{aligned} \prod_X T^{P(X)} &\leq E[T] \\ &= \sum_X TP(X) \\ \frac{1}{\prod_X X^{P(X)}} &\leq \sum_X \frac{P(X)}{X} \\ \prod_X X^{P(X)} &\geq \frac{1}{\sum_X \frac{P(X)}{X}} \\ \mathcal{G} &\geq \mathcal{H}. \end{aligned}$$

page 262, line 9: “such to the” should read “subject to the”

Thanks to Liam O’Reilly who pointed out the following errors.

The expression in the middle of page 189 for the covariance is incorrect. It should read as follows.

$$\begin{aligned} \text{Cov}(S(T)(S(T) - K)^+) &= S^2(0)e^{2(\mu+\sigma^2)T} \phi(w + 2\sigma\sqrt{T}) \\ &\quad + S(0)e^{(\mu+\sigma^2/2)T} [K\phi(w) \\ &\quad - (K + Se^{(\mu+\sigma^2/2)T}) \phi(w + \sigma\sqrt{T})] \end{aligned}$$

This of course changes Eq. (10.32) which should now read:

$$\begin{aligned} \text{Var}(R) &= e^{-2rT} \left[ a^2 S^2(0) e^{(2\mu+\sigma^2)T} (e^{\sigma^2 T} - 1) \right. \\ &\quad + b^2 \left( S^2(0) e^{2(\mu+\sigma^2)T} \phi(w + 2\sigma\sqrt{T}) - 2KS(0) e^{(\mu+\sigma^2/2)T} \phi(w + \sigma\sqrt{T}) \right. \\ &\quad \left. \left. + K^2 \phi(w) - (S(0) e^{(\mu+\sigma^2/2)T} \phi(w + \sigma\sqrt{T}) - K\phi(w))^2 \right) \right. \\ &\quad + 2ab \left( S^2(0) e^{2(\mu+\sigma^2)T} \phi(w + 2\sigma\sqrt{T}) \right. \\ &\quad + S(0) e^{(\mu+\sigma^2/2)T} [K\phi(w) \\ &\quad \left. \left. - (K + Se^{(\mu+\sigma^2/2)T}) \phi(w + \sigma\sqrt{T}) \right) \right] \end{aligned}$$

The price of the European call option in the solution to exercise 20 of Chapter 10 should be  $C = 3.33212$ . This changes several of the remaining results for that exercise. The expected return is therefore

$$E[R] = 2.23423x + 1.11282y.$$

The equation for the variance of the expected return is therefore

$$\text{Var}(R) = 114.405x^2 + 129.264xy + 46.6908y^2.$$

If we set the expected rate of return to \$10 and solve for  $y$  we obtain

$$y = 8.98622 - 2.00773x.$$

Substituting this expression into the variance of  $R$  produces the quadratic function of  $x$  given below.

$$\text{Var}(R) = 43.0877x^2 - 523.185x + 3770.38$$

The minimum value occurs when  $x = 6.07116$  and thus  $y = -3.20302$ .

Thanks to Prof. Josef Dick who pointed out the following errors.

page 164, line -2: “portfolio in minimized” should read “portfolio is minimized”.

page 169, line 2: the first line of the first displayed equation should read as follows.

$$\frac{f(y) - f(w)}{y - w} = \frac{f(y) - f(w)}{\lambda(y - x)} \leq \frac{f(y) - f(x)}{y - x}$$

page 177, line -1: as used in the example on page 178, the values on the last line of the table are not variances, but rather the standard deviations in the rates of return.

page 178, line -1: the allocation vector should be  $\langle \alpha_1, \alpha_2, \dots, \alpha_n \rangle$ .

page 260, exercise (11): it was assumed that  $r = 0.11$  though this was not stated in the exercise. The corrected, more general solution should be to let  $X$  represent the random variable of the investor’s return on the investment, then

$$X = \begin{cases} 2\alpha x + (1+r)(1-\alpha)x & \text{with probability } p, \\ (1+r)(1-\alpha)x & \text{with probability } 1-p. \end{cases}$$

If the investor’s utility function is  $u(x) = \ln x$ , then the expected value of the utility function is

$$\mathbb{E}[u(X)] = pu(2\alpha x + (1+r)(1-\alpha)x) + (1-p)u((1+r)(1-\alpha)x).$$

This function is maximized when

$$\begin{aligned} 0 &= \frac{d}{d\alpha} \mathbb{E}[u(X)] \\ &= \frac{p-1}{1-\alpha} + \frac{p(1-r)}{\alpha(1-r) + 1+r} \\ \alpha &= \frac{2p-r-1}{1-r}. \end{aligned}$$

Other errors in this chapter include the following.

page 167, Eq. (10.9) should be

$$f(\lambda x + (1 - \lambda)y) \geq \lambda f(x) + (1 - \lambda)f(y)$$

page 257, exercise (7): the first displayed equation should be as follows:

$$\begin{aligned} \ln \left( \sum_X X P(X) \right) &\geq \sum_X (\ln X) P(X) \\ &= \sum_X \ln \left( X^{P(X)} \right) \\ &= \ln \left( \prod_X X^{P(X)} \right). \end{aligned}$$

page 260, exercise (10): Prof. M.M. Chawla and Prof. Josef Dick each pointed out different errors in the solution. The displayed equation in the solution should be

$$\begin{aligned} \frac{1}{2}f(15) + \frac{1}{2}f(-15) &= f(C) \\ -\frac{225}{2} &= C - \frac{C^2}{2} \\ C &= 1 \pm \sqrt{226} \end{aligned}$$

The certainty equivalent will be the smaller of the two roots of the quadratic equation, *i.e.*,  $C = 1 - \sqrt{226} \approx -14.0333$ .