# Hypothesis Testing: $\sigma$ Unknown

## MATH 130, *Elements of Statistics I*

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# Student's $t$-Distribution

1. The $t$-distribution is different for different degrees of freedom.
2. The $t$-distribution is centered at 0 and symmetric about 0.
3. The total area under the curve is 1. The area to the left of 0 is $1/2$ and the area to the right of 0 is $1/2$.
4. As the magnitude of $t$ increases the graph approaches but never equals 0.
5. The area in the tails of the $t$-distribution is larger than the area in the tails of the normal distribution.
6. As sample size $n$ increases, the distribution becomes approximately normal.

# Testing Hypotheses: $\sigma$ Unknown

1. Determine $H_0$ and $H_1$.
2. Select a level of significance $\alpha$.
3. Compute test statistic $t_0 = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$.

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4. (Classical Approach) Determine the critical value(s) of $t$
5. Compare the critical value with the test statistic.

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4. (*P*-Value Approach) Determine the *P*-value.
5. If the *P*-value $< \alpha$, reject the null hypothesis.

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6. State the conclusion.

## Example

A nutritionist believes that children under the age of 10 years are consuming more than the US Food and Drug Administration's recommended daily allowance of 2400 mg of sodium. She obtains a random sample of 75 children under the age of 10 and measures their daily consumption of sodium. The sample mean is 2993 mg, with a sample standard deviation of 1489 mg. Is there significant evidence to conclude that children under the age of 10 years are consuming too much sodium? Use the $\alpha = 0.05$ level of significance.

# Example (2 of 3, Classical Approach)

$H_0$: $\mu = 2400$

$H_1$: $\mu > 2400$ (right-tailed test)

$$\alpha = 0.05, \quad df = 74, \quad t_\alpha = 1.667$$

$$\text{Test statistic:} \quad t_0 = \frac{2993 - 2400}{1489/\sqrt{75}} = 3.449$$

# Example (3 of 3)

**Decision:** reject $H_0$.

**Conclusion:** the sample data supports the claim that children under the age of 10 years are consuming more than the recommended 2400 mg of sodium daily.

# Example (1 of 3)

## Example

In 2001, the mean household expenditure for energy was \$1493, according to data obtained from the US Energy Information Administration. An economist wanted to know whether this amount has changed significantly since 2001. In a random sample of 35 households, he found that the mean expenditure (adjusted to 2001 dollars) for energy was \$1618 with a standard deviation of \$321. At the $\alpha = 0.05$ level of significance, has the mean expenditure for energy changed significantly since 2001?

# Example (2 of 3, Classical Approach)

$H_0$: $\mu = 1493$

$H_1$: $\mu \neq 1493$ (two-tailed test)

$$\alpha = 0.05, \quad \alpha/2 = 0.025, \quad df = 34, \quad t_{\alpha/2} = \pm 2.032$$

$$\text{Test statistic:} \quad t_0 = \frac{1618 - 1493}{321/\sqrt{35}} = 2.304$$

**Decision:** reject $H_0$.

**Conclusion:** the sample data supports the claim that the mean expenditure for energy changed significantly since 2001.

## Example

The USGA requires that golf balls have a weight that is less than 1.62 ounces. An engineer for the USGA wants to verify that Maxfli XS balls conform to USGA standards. She obtains a random sample of 12 Maxfli XS golf balls and measures the weights.

| | | | | | |
|---|---|---|---|---|---|
| 1.614 | 1.619 | 1.614 | 1.614 | 1.610 | 1.610 |
| 1.621 | 1.612 | 1.615 | 1.621 | 1.602 | 1.617 |

Decide whether the golf balls meet the USGA's standard at the $\alpha = 0.10$ level.

# Example (2 of 3, *P*-Value Approach)

$H_0$: $\mu = 1.62$

$H_1$: $\mu < 1.62$ (left-tailed test)

$$\alpha = 0.10, \quad df = 11, \quad t_\alpha = -1.363$$

Test statistic: $$t_0 = \frac{1.6141 - 1.62}{0.0053/\sqrt{12}} = -3.843$$

# Example (3 of 3)

$$\begin{aligned} -4.025 &< t_0 = -3.843 < -3.497 \\ \implies \quad 0.0025 &< P\text{-value} < 0.001 \end{aligned}$$

**Decision:** reject $H_0$.

**Conclusion:** the sample data supports the claim that the Maxfli XS golf balls meet the USGA standard.

## Example

The city has hired a new contractor to replace burned-out streetlights. The previous contractor replaced streetlights in an average of 3.2 days. The city council believes the new contractor is not getting the streetlights replaced as quickly. In a random sample of 12 streetlight service calls the replacement times were (in days):

| | | | | | |
|---|---|---|---|---|---|
| 6.2 | 7.1 | 5.4 | 5.5 | 7.5 | 2.6 |
| 4.3 | 2.9 | 3.7 | 0.7 | 5.6 | 1.7 |

Is there evidence that the new contractor is not performing up to the standard set by the previous contractor? Use the $\alpha = 0.05$ level of significance.

# Example (2 of 3, $P$-Value Approach)

$H_0$: $\mu = 3.2$

$H_1$: $\mu > 3.2$ (right-tailed test)

$$\alpha = 0.05, \quad df = 11, \quad t_\alpha = 1.796$$

$$\text{Test statistic:} \quad t_0 = \frac{4.43 - 3.2}{2.15/\sqrt{12}} = 1.99$$

# Example (3 of 3)

$$\begin{aligned} 1.796 &< t_0 = 1.99 < 2.201 \\ \implies \quad 0.025 &< P\text{-value} < 0.05 \end{aligned}$$

**Decision:** reject $H_0$.

**Conclusion:** the sample data supports the claim that the new contractor is not performing up to the standard set by the previous contractor.

# Homework

- Read Section 10.3.
- Pages 487-491: 5, 9, 13, 17, 21, 25, 29