Millersville University   **Name** **Answer Key**
Department of Mathematics
MATH 130, *Elements of Statistics I*, Homework 6
December 10, 2009

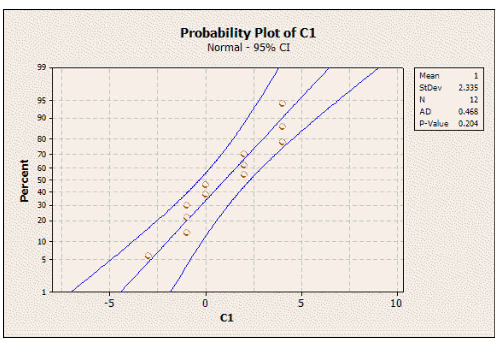
Page 478, Exercise 28

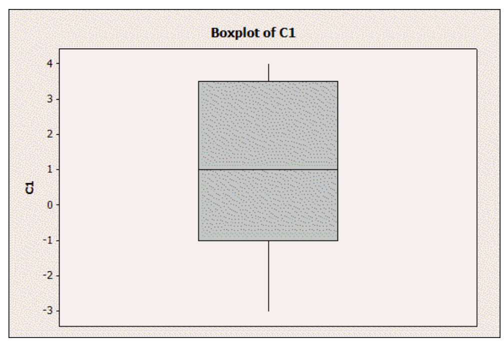
It's a Hot One! Recently, a friend of main claimed that the summer of 2000 in Houston, Texas, was hotter than usual, To test his claim, I went to AccuWeather.com and randomly selected 12 days in the summer of 2000. I then recorded the departure from normal, with positive values indicating above-normal temperatures and negative values indicating below-normal temperatures, as shown in the following:

$+4$	-1	0	$+2$
$+2$	$+4$	-1	-3
-1	0	$+2$	$+4$

Source: AccuWeather.com

(a) Because the sample size is small, I must verify that the temperature departure is normally distributed and the sample does not contain any outliers. The normal probability plot and boxplot are shown. Are the conditions for testing the hypothesis satisfied?





Since the data appear to come from a normal distribution, the hypothesis may be tested.

(b) Was the summer of 2000 hotter than normal in Houston? Assume that $\sigma = 1.8$. Use the $\alpha = 0.05$ level of significance.

The null and alternative hypotheses are respectively

H_0: $\mu = 0$

H_1: $\mu > 0$ (right-tailed test),

where μ represents the population departure from normal temperature. The critical value of $z_\alpha = z_{0.05} = 1.645$. If X represents the sample departure from normal temperature, then $\overline{X} = 1.0$. The test statistic is

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{1.0 - 0.0}{1.8/\sqrt{12}} = 1.925.$$

- If we make the decision using the classical approach, since the test statistic falls in the critical region, we reject the null hypothesis.
- If we make the decision using the p-value approach, then

  $$p = P(Z > 1.925) = 1 - P(Z \leq 1.925) = 1 - 0.9726 = 0.0274 < \alpha = 0.05$$

  and likewise we reject the null hypothesis.

Conclusion: the sample data support the friend's claim that the summer of 2000 in Houston was hotter than normal.

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Normal Temperature Carl Reinhold August Wunderlich said that the mean temperature of humans is 98.6°F. Researchers Philip Mackowiak, Steven Wasserman, and Myron Levine [*JAMA*, Sept. 23-30 1992; 268(12):1578-80] measured the temperatures of 26 females 1 to 4 times daily for 3 days to get a total of 123 measurements. The sample data yielded a sample mean of 98.4°F and a sample standard deviation of 0.7°F.

(a) Using the classical approach, judge whether the normal temperature of women is less than 98.6°F at the $\alpha = 0.01$ level of significance.

The null and alternative hypotheses are respectively

H_0: $\mu = 98.6$°F

H_1: $\mu < 98.6$°F (left-tailed test),

where μ represents the population mean temperature for women. The critical value of t for $\alpha = 0.01$ level of significance and $df = 122 \approx 100$ is $t = -2.364$. The value of the test statistic is

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{98.4 - 98.6}{0.7/\sqrt{123}} = -3.169.$$

Since the test statistic falls in the critical region, the null hypothesis is rejected.

Conclusion: the sample data support the claim that the mean temperature of women is less than 98.6°F.

(b) Determine and interpret the p-value.

Examining again the row of Table V (t-distribution) with $df = 100$, the test statistic falls between -3.174 and -2.871. Thus $0.001 < p < 0.0025$. The probability of obtaining a sample with a mean as extreme as or more extreme than the one the researchers got is the p-value.

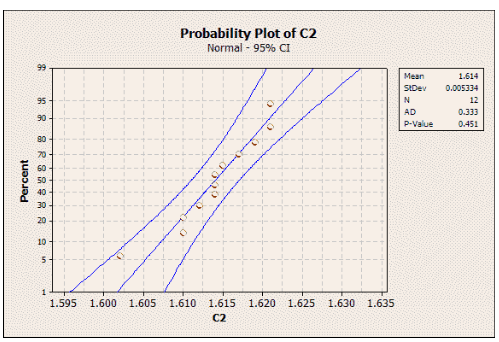
Page 489, Exercise 22

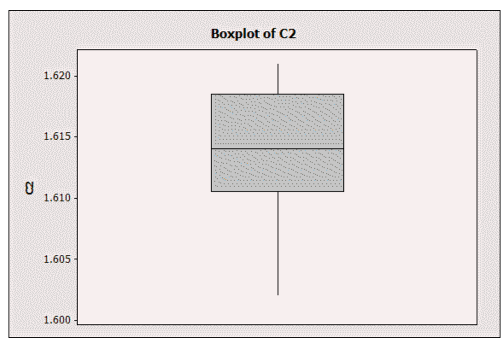
Conforming Golf Balls The USGA requires that golf balls have a weight that is less than 1.62 ounces. An engineer for the USGA wants to verify that Maxfli XS balls conform to USGA standards. He obtains a random sample of 12 Maxfli XS golf balls and obtains the weights in the table.

1.614	1.619	1.614
1.614	1.610	1.610
1.621	1.612	1.615
1.621	1.602	1.617

Source: Michael McCraith, Joliet Junior College

(a) Because the sample is small, the engineer must verify that weight is normally distributed and that the sample does not contain any outliers. The normal probability plot and boxplot are shown. Are the conditions for testing the hypothesis satisfied?





Since the data appear to come from a normal distribution, the hypothesis may be tested.

(b) Decide whether the golf balls meet Maxfli's standard at the $\alpha = 0.10$ level.

The null and alternative hypotheses are respectively

H_0: $\mu = 1.62$ oz.

H_1: $\mu < 1.62$ oz. (left-tailed test), where μ represents the population mean weight of Maxfli XS golf balls. The critical value of t for $\alpha = 0.10$ level of significance and $df = 11$ is $t_\alpha = -1.363$. The sample mean and sample standard deviation are $\overline{x} = 1.6141$ and $s = 0.0053$ respectively.

The value of the test statistic is

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{1.6141 - 1.62}{0.0053/\sqrt{12}} = -3.856.$$

Since the test statistic falls in the critical region, the null hypothesis is rejected.

Conclusion: the sample data support the claim that Maxfli XS golf balls weigh less than 1.62 ounces.