Please answer the following questions. Your answers will be evaluated on their correctness, completeness, and use of statistical concepts we have covered. Please show all work and write out your work neatly. Answers without supporting work will receive no credit. The point values of the problems are listed in parentheses.

1. (4 points each) According to the 1993 *World Factbook*, the 1993 total fertility rate (mean number of children born per woman) for Madagascar is 6.7. Suppose the standard deviation of the total fertility rate is 2.5. Suppose samples of size $n = 200$ are collected and the mean number of children per woman is calculated for each sample.

(a) What is the mean of the sample means?

(b) What is the standard deviation of the sample means?

(c) Describe the shape of the distribution of the sample means.
2. (10 points) According the U.S. Department of Energy, the average price of gasoline is $2.13 per gallon. The price is normally distributed with a standard deviation of $0.12 per gallon. A random sample of the price per gallon at 35 gasoline stations is collected. Find the probability that the sample mean price per gallon is between $2.10 and $2.17.

3. (10 points) According to an article in *Pharmaceutical News*, a sample of 45 people aged 65 or older spent an average of $75 per month on medication. The standard deviation for the sample was $30. Construct the 99% confidence interval estimate for the average monthly amount spent on medication by all people aged 65 or older.
4. (3 points each) Suppose that a hypothesis test is being conducted. In each statement below some information on an hypothesis test is given. The significance level \( \alpha \), the type of test (left-tailed, right-tailed, or two-tailed), the test statistic (\( t^* \) or \( z^* \)), and where appropriate the number of degrees of freedom are stated. Under each set of circumstances determine whether the null hypothesis of the test would be rejected or would fail to be rejected. Justify your answers.

(a) \( \alpha = 0.05, z^* = 1.83 \), right-tailed test

(b) \( \alpha = 0.02, t^* = 2.60, \text{ df } = 20 \), two-tailed test

(c) \( \alpha = 0.05, z^* = -2.45 \), two-tailed test

(d) \( \alpha = 0.10, t^* = -1.27, \text{ df } = 10 \), left-tailed test
5. (10 points) The Friends of the Ganser library are trying to determine how many times per semester students visit the library. Suppose that the standard deviation in the number of times a student visits the library has been estimated to be $\sigma = 10.6$. How large a sample should be collected so that the margin of error at the 98% confidence level in the estimation of the population mean number of visits to the library is no greater than 3?

6. (12 points) According to an article in *Good Housekeeping* a 138-lb woman who walks for 30 minutes, 4 times per week at a steady 4-mph pace can lose up to 10 lb over the span of one year. Suppose 60 women with weights between 135 and 145 lbs followed this plan for one year. At the end of the year the average weight loss for the sample was 8.9 lbs. The standard deviation of the weight loss was 3.9 lbs. Carry out a hypothesis test of the claim that the mean amount of weight lost by all women following the plan is 10 lbs. Use the 95% level of significance. Clearly state the final conclusion.
7. (12 points) An insurance company felt that the mean distance from homes in a particular neighborhood to the nearest fire department was at least 4.8 miles. Homeowners in the neighborhood set out to show that the distance was less than 4.8 miles. If they were successful in doing this, the insurance company would offer them insurance at a lower cost. In a random sample of 40 homes in the neighborhood the mean distance to the nearest fire department was 4.5 miles. If the standard deviation in the distance from all homes to the nearest fire department is $\sigma = 2.3$ miles, does this sample provide sufficient evidence to support the neighborhood’s claim at the $\alpha = 0.04$ level of significance? You must justify your answer.
8. (10 points) A natural gas utility company is considering purchasing new tires for its fleet of service vehicles. The decision will be based on the expected mileage of the tires. For a sample of 20 tires tested, the mean mileage was 35,000 and the standard deviation was 2500 miles. Construct the 95% confidence interval estimate of the mean mileage the utility company should expect for its entire fleet.
9. (12 points) In a large cherry orchard the average yield has been 4.53 tons per acre for the last several years. A new fertilizer is tested on 12 randomly selected one-acre plots. The average yield from these plots was 4.89 tons per acre with standard deviation of 0.35 tons per acre. Assuming the yield per acre is normally distributed, at the 0.01 level of significance, is there sufficient evidence to claim an increase in the yield per acre with the new fertilizer? Justify your answer.