Millersville University
Department of Mathematics
MATH 161, *Calculus I*

Please evaluate the following indefinite integrals.

1. $\int 4x^3\,dx$

$$\begin{aligned}\int 4x^3\,dx &= 4\int x^3\,dx \\ &= 4\frac{x^{3+1}}{3+1}+C \\ &= 4\frac{x^4}{4}+C \\ &= x^4+C\end{aligned}$$

2. $\int 2x^3+3x^2\,dx$

$$\begin{aligned}\int 2x^3+3x^2\,dx &= \int 2x^3\,dx+\int 3x^2\,dx \\ &= 2\int x^3\,dx+3\int x^2\,dx \\ &= 2\frac{x^{3+1}}{3+1}+3\frac{x^{2+1}}{2+1}+C \\ &= 2\frac{x^4}{4}+3\frac{x^3}{3}+C \\ &= \frac{1}{2}x^4+x^3+C\end{aligned}$$

3. $\int x^{2/3}+3x^{1/3}+4x^2\,dx$

$$\begin{aligned}\int x^{2/3}+3x^{1/3}+4x^2\,dx &= \int x^{2/3}\,dx+\int 3x^{1/3}\,dx+\int 4x^2\,dx \\ &= \int x^{2/3}\,dx+3\int x^{1/3}\,dx+4\int x^2\,dx \\ &= \frac{x^{1+2/3}}{1+2/3}+3\frac{x^{1+1/3}}{1+1/3}+4\frac{x^{1+2}}{1+2}+C \\ &= \frac{x^{5/3}}{5/3}+3\frac{x^{4/3}}{4/3}+4\frac{x^3}{3}+C \\ &= \frac{3}{5}x^{5/3}+\frac{9}{4}x^{4/3}+\frac{4}{3}x^3+C\end{aligned}$$

4. $\int \sin x - x \, dx$

$$\begin{aligned}\int \sin x - x \, dx &= \int \sin x \, dx - \int x \, dx \\ &= -\cos x - \frac{x^{1+1}}{1+1} + C \\ &= -\cos x - \frac{x^2}{2} + C\end{aligned}$$

5. $\int 3x(x^3+1) \, dx$

$$\begin{aligned}\int 3x(x^3+1) \, dx &= 3\int x(x^3+1) \, dx \\ &= 3\int (x^4+x) \, dx \\ &= 3\left(\int x^4 \, dx + \int x \, dx\right) \\ &= 3\left(\frac{x^{4+1}}{4+1} + \frac{x^{1+1}}{1+1}\right) + C \\ &= 3\left(\frac{x^5}{5} + \frac{x^2}{2}\right) + C \\ &= \frac{3}{5}x^5 + \frac{3}{2}x^2 + C\end{aligned}$$

6. $\int \cot^2 x \, dx$

$$\begin{aligned}\int \cot^2 x \, dx &= \int \frac{\cos^2 x}{\sin^2 x} \, dx \\ &= \int \frac{1-\sin^2 x}{\sin^2 x} \, dx \\ &= \int \frac{1}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x} \, dx \\ &= \int \csc^2 x - 1 \, dx \\ &= \int \csc^2 x \, dx - \int 1 \, dx \\ &= -\cot x - x + C\end{aligned}$$

7. $\int \sqrt{x} + \cos x \, dx$

$$
\begin{aligned}
\int \sqrt{x} + \cos x \, dx &= \int x^{1/2} + \cos x \, dx \\
&= \int x^{1/2} \, dx + \int \cos x \, dx \\
&= \frac{x^{1+1/2}}{1+1/2} + \sin x + C \\
&= \frac{x^{3/2}}{3/2} + \sin x + C \\
&= \frac{2}{3} x^{3/2} + \sin x + C
\end{aligned}
$$

8. $\int \frac{x^{1/2} + x^3 - x^4}{x} \, dx$

$$
\begin{aligned}
\int \frac{x^{1/2} + x^3 - x^4}{x} \, dx &= \int \frac{x^{1/2}}{x} + \frac{x^3}{x} - \frac{x^4}{x} \, dx \\
&= \int x^{-1/2} + x^2 - x^3 \, dx \\
&= \int x^{-1/2} \, dx + \int x^2 \, dx - \int x^3 \, dx \\
&= \frac{x^{1-1/2}}{1-1/2} + \frac{x^{1+2}}{1+2} - \frac{x^{1+3}}{1+3} + C \\
&= \frac{x^{1/2}}{1/2} + \frac{x^3}{3} - \frac{x^4}{4} + C \\
&= 2x^{1/2} + \frac{x^3}{3} - \frac{x^4}{4} + C
\end{aligned}
$$

9. $\int (x^3 - x^5)^2 \, dx$

$$
\begin{aligned}
\int (x^3 - x^5)^2 \, dx &= \int (x^3 - x^5)(x^3 - x^5) \, dx \\
&= \int x^6 - 2x^8 + x^{10} \, dx \\
&= \int x^6 \, dx - \int 2x^8 \, dx + \int x^{10} \, dx \\
&= \int x^6 \, dx - 2 \int x^8 \, dx + \int x^{10} \, dx \\
&= \frac{x^{6+1}}{6+1} - 2\frac{x^{8+1}}{8+1} + \frac{x^{10+1}}{10+1} + C
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x^7}{7} - 2\frac{x^9}{9} + \frac{x^{11}}{11} + C \\
&= \frac{x^7}{7} - \frac{2}{9}x^9 + \frac{x^{11}}{11} + C
\end{aligned}
$$

10. $\int x^3(x+1)(3x+2)\,dx$

$$
\begin{aligned}
\int x^3(x+1)(3x+2)\,dx &= \int x^3[(x+1)(3x+2)]\,dx \\
&= \int x^3[3x^2+5x+2]\,dx \\
&= \int 3x^5+5x^4+2x^3\,dx \\
&= \int 3x^5\,dx + \int 5x^4\,dx + \int 2x^3\,dx \\
&= 3\int x^5\,dx + 5\int x^4\,dx + 2\int x^3\,dx \\
&= 3\frac{x^{5+1}}{5+1} + 5\frac{x^{4+1}}{4+1} + 2\frac{x^{3+1}}{3+1} + C \\
&= 3\frac{x^6}{6} + 5\frac{x^5}{5} + 2\frac{x^4}{4} + C \\
&= \frac{1}{2}x^6 + x^5 + \frac{1}{2}x^4 + C
\end{aligned}
$$

11. $\int e^{-2x}\,dx$

$$
\begin{aligned}
\int e^{-2x}\,dx &= \frac{1}{-2}e^{-2x} + C \\
&= -\frac{1}{2}e^{-2x} + C
\end{aligned}
$$

12. $\int 4\sin(3x)\,dx$

$$
\begin{aligned}
\int 4\sin(3x)\,dx &= 4\int \sin(3x)\,dx \\
&= 4\cdot\frac{1}{3}(-\cos(3x)) + C \\
&= -\frac{4}{3}\cos(3x) + C
\end{aligned}
$$

13. $\displaystyle\int \frac{3x^2}{x^3+2}\,dx$

$$\int \frac{3x^2}{x^3+2}\,dx = \ln|x^3+2| + C$$

14. $\displaystyle\int \frac{e^x}{e^x+1}\,dx$

$$\begin{aligned}\int \frac{e^x}{e^x+1}\,dx &= \ln|e^x+1| + C \\ &= \ln(e^x+1) + C\end{aligned}$$