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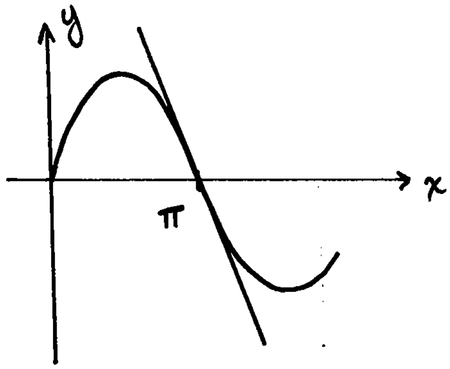
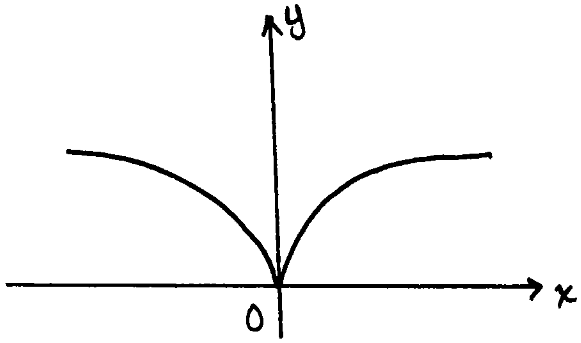
5)



7)



Due to the presence of the cusp at $x=0$, there is no well-defined tangent line at $x=0$.

9) $m \approx -1$

11) C B A D

13) $f(x) = x^3 - x$

a) $m_s = \dfrac{f(2)-f(1)}{2-1} = 6$

b) $m_s = \dfrac{f(3)-f(2)}{3-2} = 18$

c) $m_s = \dfrac{f(2)-f(1.5)}{2-1.5} = 8.25$

d) $m_s = \dfrac{f(2.5)-f(2)}{2.5-2} = 14.25$

e) $m_s = \dfrac{f(2)-f(1.9)}{2-1.9} = 10.41$

f) $m_s = \dfrac{f(2.1)-f(2)}{2.1-2} = 11.61$

g) $m \approx 11$

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21) $f(x) = x^2 - 2$, $a = 1$

$$m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{(1+h)^2 - 2 - (1^2 - 2)}{h}$$

$$= \lim_{h \to 0} \frac{1 + 2h + h^2 - 2 + 1}{h}$$

$$= \lim_{h \to 0} \frac{2h + h^2}{h}$$

$$= \lim_{h \to 0} (2 + h)$$

$$= 2$$

Point-slope formula: $2 = \dfrac{y - f(1)}{x - 1}$

$$2(x-1) = y + 1$$

$y = 2x - 3$ tangent line

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23) $f(x) = x^2 - 3x$, $a = -2$

$$m = \lim_{h \to 0} \frac{f(-2+h) - f(-2)}{h} = \lim_{h \to 0} \frac{(-2+h)^2 - 3(-2+h) - 10}{h}$$

$$= \lim_{h \to 0} \frac{4 - 4h + h^2 + 6 - 3h - 10}{h}$$

$$= \lim_{h \to 0} \frac{-7h + h^2}{h}$$

$$= \lim_{h \to 0} (-7 + h)$$

$$= -7$$

Point-slope formula: $-7 = \dfrac{y - f(-2)}{x - (-2)}$

$$-7(x+2) = y - 10$$

$$-7x - 4 = y$$ tangent line

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25) $f(x) = \dfrac{2}{x+1}$, $a = 1$

$$m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2}{1+h+1} - 1}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2}{2+h} - 1}{h} \cdot \frac{(2+h)}{(2+h)}$$

$$= \lim_{h \to 0} \frac{2 - (2+h)}{h(2+h)}$$

$$= \lim_{h \to 0} \frac{-h}{h(2+h)}$$

$$= \lim_{h \to 0} \frac{-1}{2+h}$$

$$m = -1/2$$

Point-slope formula: $-1/2 = \dfrac{y - f(1)}{x-1}$

$$-\frac{1}{2}(x-1) = y - 1$$

$$-\frac{1}{2}x + \frac{3}{2} = y$$ tangent line

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27) $f(x) = \sqrt{x+3}$, $a = -2$

$$m = \lim_{h\to 0} \frac{f(-2+h) - f(-2)}{h} = \lim_{h\to 0} \frac{\sqrt{-2+h+3} - 1}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{1+h} - 1}{h} \cdot \frac{(\sqrt{1+h} + 1)}{(\sqrt{1+h} + 1)}$$

$$= \lim_{h\to 0} \frac{1+h-1}{h(\sqrt{1+h}+1)}$$

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{1+h}+1)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1}$$

$m = 1/2$

Point-slope formula: $\frac{1}{2} = \frac{y - f(-2)}{x-(-2)}$

$$\frac{1}{2}(x+2) = y - 1$$

$$\frac{1}{2}x + 2 = y \quad \text{tangent line}$$

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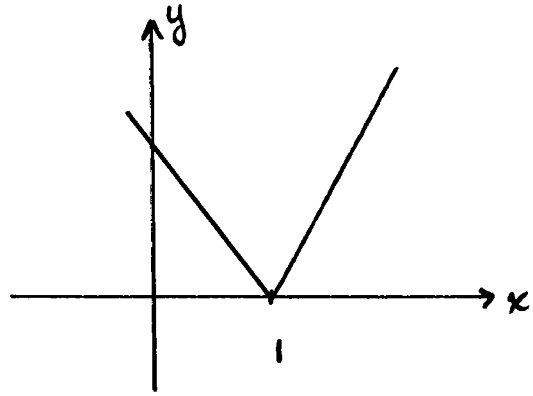
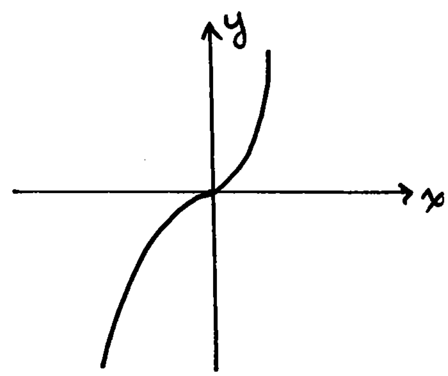
29) $f(x) = |x-1|, \quad a = 1$



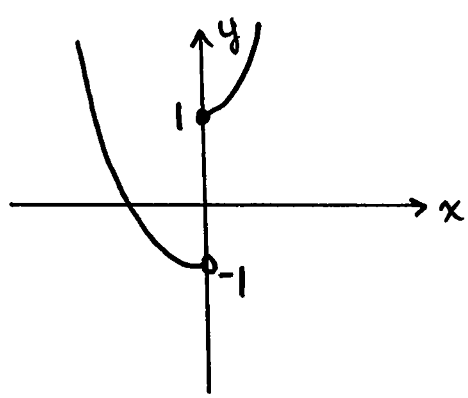
The tangent line does not exist at $x=1$ because of the corner in the graph.

31) $f(x) = \begin{cases} -2x^2 & \text{if } x<0 \\ x^3 & \text{if } x \geq 0 \end{cases}$ at $a = 0$



The slope of the tangent line at $a=0$ is $m=0$.

33) $f(x) = \begin{cases} x^2-1 & \text{if } x<0 \\ x^3+1 & \text{if } x \geq 0 \end{cases}$ at $a=0$



The tangent line does not exist at $x=0$ because of the jump discontinuity.

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35) $f(t) = 16t^2 + 10$

a) $v_a = \dfrac{f(2) - f(0)}{2-0} = 32$

b) $v_a = \dfrac{f(2) - f(1)}{2-1} = 48$

c) $v_a = \dfrac{f(2) - f(1.9)}{2-1.9} = 62.4$

d) $v_a = \dfrac{f(2) - f(1.99)}{2-1.99} = 63.84$

e) $v \approx 64$

39) $f(t) = -16t^2 + 5, \quad a = 1$

$$v = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{-16(1+h)^2 + 5 - (-16+5)}{h}$$

$$= \lim_{h \to 0} \frac{-16(1 + 2h + h^2) + 5 + 16 - 5}{h}$$

$$= \lim_{h \to 0} \frac{-32h - 16h^2}{h}$$

$$= \lim_{h \to 0} (-32 - 16h)$$

$$v = -32$$

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41) $f(t) = \sqrt{t+16}$, $a = 0$

$$v = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{f(h) - 4}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{h+16} - 4}{h} \cdot \frac{(\sqrt{h+16} + 4)}{(\sqrt{h+16} + 4)}$$

$$= \lim_{h \to 0} \frac{h+16-16}{h(\sqrt{h+16} + 4)}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{h+16}+4)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{h+16} + 4}$$

$$v = 1/8$$

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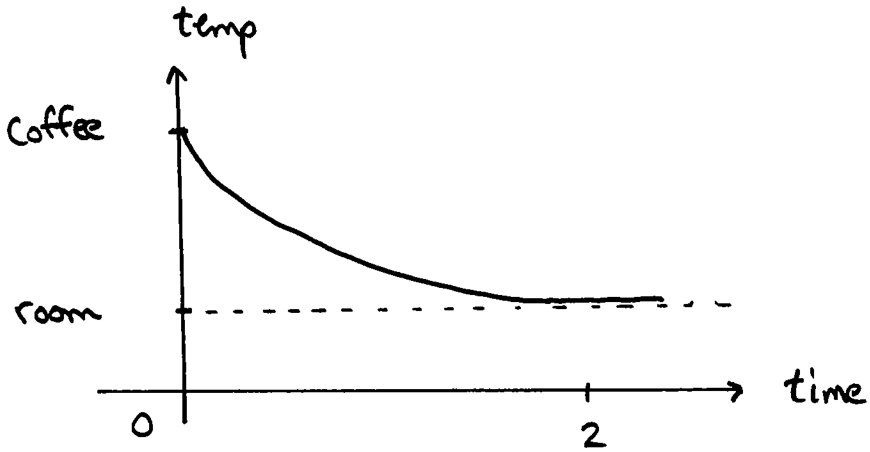
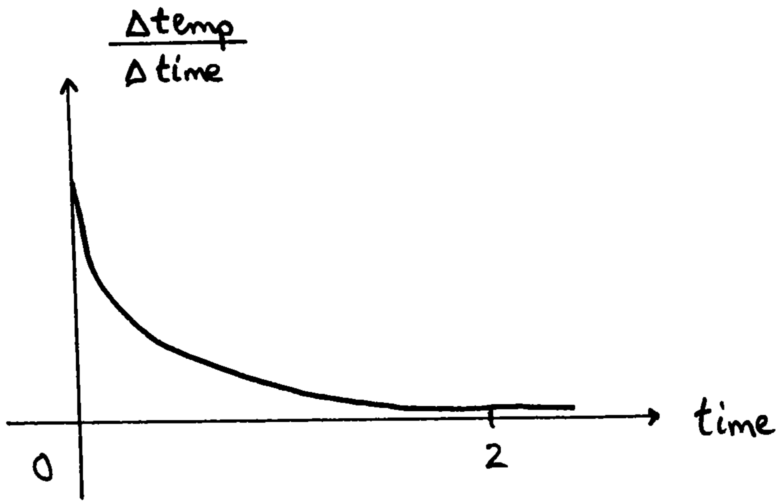
45)





53) $$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{(x-a) \to 0} \frac{f(a+(x-a)) - f(a)}{(x-a)}$$ if we let $h = x-a$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{x-a}.$$