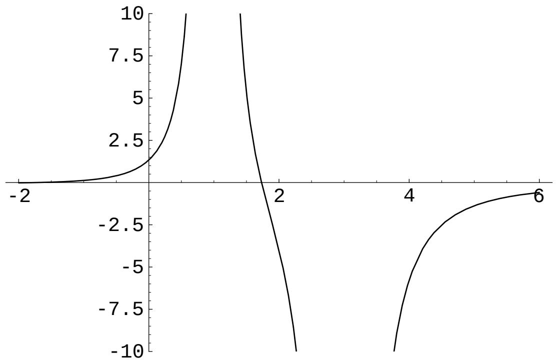
Comprehensive Graphing

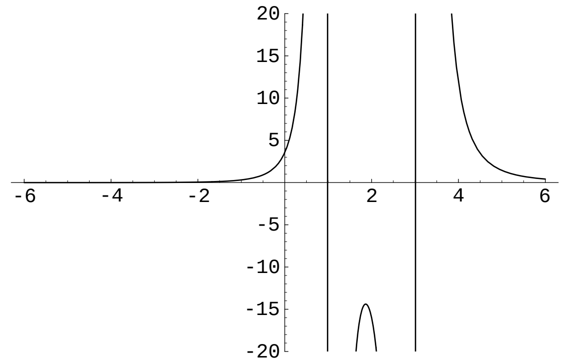
1 Example 1

Suppose we have the following function and its first and second derivatives:

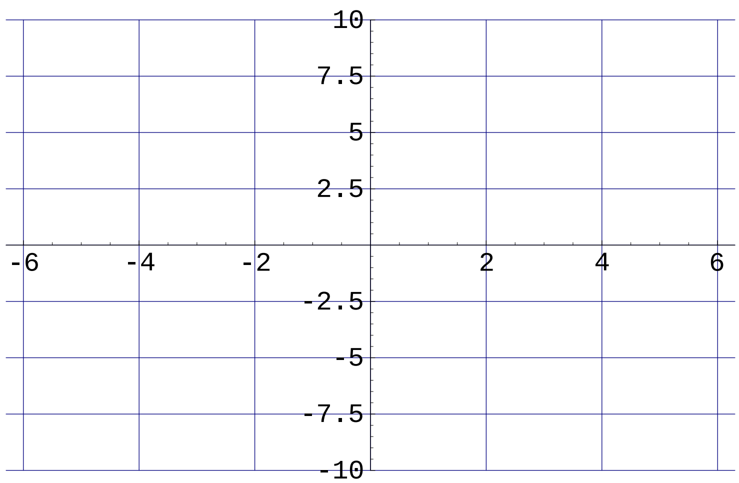
$$f(x) = \frac{4x}{x^2 - 4x + 3}$$
$$f'(x) = \frac{4(3 - x^2)}{(x^2 - 4x + 3)^2}$$
$$f''(x) = \frac{8(x^3 - 9x + 12)}{(x^2 - 4x + 3)^3}$$

The graphs of the first and second derivatives resemble:





We can estimate from the graph of the first derivative or solve algebraically from the formula for the first derivative that $f'(\pm\sqrt{3}) = 0$. It is very difficult to see from the graph of the first derivative that $f'(x) < 0$ for $x < -\sqrt{3}$. It is very difficult to see from the graph of the second derivative that $f''(x) < 0$ for $x < -3.522$. If $f(-\sqrt{3}) \approx -0.536$, $f(\sqrt{3}) \approx -7.464$, and $f(-3.522) \approx -0.478$ sketch the graph of $f(x)$.

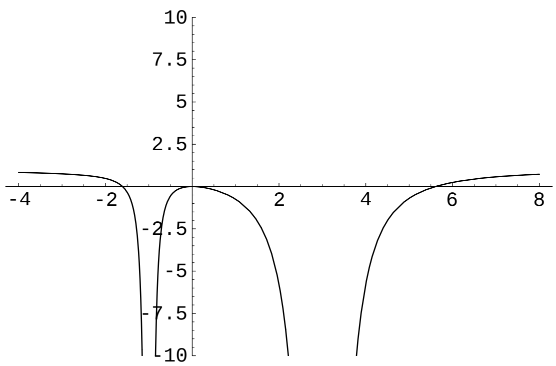


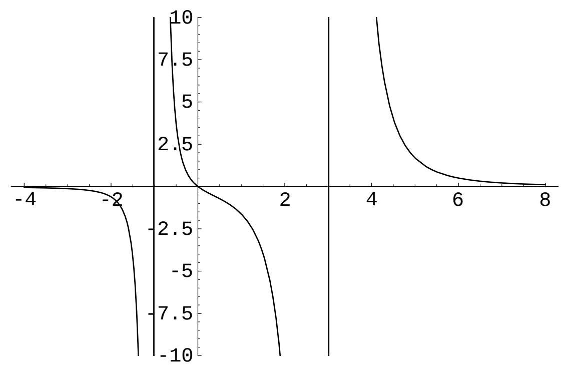
2 Example 2

Suppose we have the following function and its first and second derivatives:

$$
\begin{aligned}
f(x) &= \frac{x^3}{x^2 - 2x - 3} \\
f'(x) &= \frac{x^2(x^2 - 4x - 9)}{(x^2 - 2x - 3)^2} \\
f''(x) &= \frac{2x(7x^2 + 18x + 27)}{(x^2 - 2x - 3)^3}
\end{aligned}
$$

The graphs of the first and second derivatives resemble:





We can estimate from the graph of the first derivative or solve algebraically from the formula for the first derivative that $f'(0) = 0$, $f'(2 - \sqrt{13}) = 0$, and $f'(2 + \sqrt{13}) = 0$. The second derviative is equal to zero only at $x = 0$. If $f(0) = 0$, $f(2 - \sqrt{13}) \approx -1.48402$, and $f(2 + \sqrt{13}) \approx 10.234$ sketch the graph of $f(x)$.