Millersville University
Mathematics Department
MATH 161, Integration by Substitution Practice

Use the Method of Substitution to evaluate the following definite and indefinite integrals.

1. $\int 2x(x^2+3)^4\,dx$ Let

$$\begin{aligned} u &= x^2+3 \\ du &= 2x\,dx \end{aligned}$$

then

$$\begin{aligned} \int 2x(x^2+3)^4\,dx &= \int u^4\,du \\ &= \frac{1}{5}u^5 + C \\ &= \frac{1}{5}(x^2+3)^5 + C \end{aligned}$$

2. $\int_0^1 x^2(1+2x^3)^5\,dx$ Let

$$\begin{aligned} u &= 1+2x^3 \\ du &= 6x^2\,dx \\ \frac{1}{6}du &= x^2\,dx \end{aligned}$$

then

$$\begin{aligned} \int_0^1 x^2(1+2x^3)^5\,dx &= \int_1^3 \frac{1}{6}u^5\,du \\ &= \frac{1}{6}\int_1^3 u^5\,du \\ &= \frac{1}{6}\cdot\frac{1}{6}\,u^6\Big|_1^3 \\ &= \frac{1}{36}\,u^6\Big|_1^3 \\ &= \frac{1}{36}\left(3^6-1^6\right) \\ &= \frac{182}{9} \approx 20.222 \end{aligned}$$

3. $\int \sec x \tan x \sqrt{1+\sec x}\,dx$ Let

$$\begin{aligned} u &= 1+\sec x \\ du &= \sec x \tan x\,dx \end{aligned}$$

then

$$\begin{aligned}\int \sec x \tan x \sqrt{1+\sec x}\, dx &= \int \sqrt{u}\, du \\ &= \int u^{1/2}\, du \\ &= \frac{2}{3} u^{3/2} + C \\ &= \frac{2}{3}(1+\sec x)^{3/2} + C\end{aligned}$$

4. $\displaystyle\int_3^4 \frac{1}{(x-2)^3}\, dx$ Let

$$\begin{aligned}u &= x-2 \\ du &= dx\end{aligned}$$

then

$$\begin{aligned}\int_3^4 \frac{1}{(x-2)^3}\, dx &= \int_1^2 \frac{1}{u^3}\, du \\ &= \int_1^2 u^{-3}\, du \\ &= -\frac{1}{2}\, u^{-2}\Big|_1^2 \\ &= -\frac{1}{2}\, \frac{1}{u^2}\Big|_1^2 \\ &= -\frac{1}{2}\left(\frac{1}{2^2} - \frac{1}{1^2}\right) \\ &= -\frac{1}{2}\left(\frac{1}{4} - 1\right) \\ &= \frac{3}{8} = 0.375\end{aligned}$$

5. $\displaystyle\int x^2 \sin(1-x^3)\, dx$ Let

$$\begin{aligned}u &= 1-x^3 \\ du &= -3x^2\, dx \\ -\frac{1}{3}du &= x^2\, dx\end{aligned}$$

then

$$\begin{aligned}\int x^2 \sin(1-x^3)\, dx &= \int -\frac{1}{3} \sin u\, du \\ &= -\frac{1}{3} \int \sin u\, du\end{aligned}$$

$$\begin{aligned} &= -\frac{1}{3}(-\cos u) + C \\ &= \frac{1}{3}\cos u + C \\ &= \frac{1}{3}\cos(1 - x^3) + C \end{aligned}$$

6. $\displaystyle\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx$ Let

$$\begin{aligned} u &= x^2 \\ du &= 2x\,dx \\ \frac{1}{2}du &= x\,dx \end{aligned}$$

then

$$\begin{aligned} \int_0^{\sqrt{\pi}} x\cos(x^2)\,dx &= \int_0^{\pi} \frac{1}{2}\cos u\,du \\ &= \frac{1}{2}\int_0^{\pi} \cos u\,du \\ &= \frac{1}{2}\sin u|_0^{\pi} \\ &= \frac{1}{2}(\sin\pi - \sin 0) \\ &= \frac{1}{2}(0 - 0) \\ &= 0 \end{aligned}$$

7. $\displaystyle\int e^x\sqrt{1+e^x}\,dx$ Let

$$\begin{aligned} u &= 1 + e^x \\ du &= e^x\,dx \end{aligned}$$

then

$$\begin{aligned} \int e^x\sqrt{1+e^x}\,dx &= \int \sqrt{u}\,du \\ &= \int u^{1/2}\,du \\ &= \frac{2}{3}u^{3/2} + C \\ &= \frac{2}{3}(1+e^x)^{3/2} + C \end{aligned}$$

8. $\displaystyle\int_1^4 \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx$ Let

$$u = 1 + \frac{1}{x}$$

$$
\begin{aligned}
du &= -\frac{1}{x^2}\,dx \\
-du &= \frac{1}{x^2}\,dx
\end{aligned}
$$

then

$$
\begin{aligned}
\int_1^4 \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx &= \int_2^{5/4} -\sqrt{u}\,du \\
&= -\int_2^{5/4} \sqrt{u}\,du \\
&= \int_{5/4}^2 \sqrt{u}\,du \\
&= \int_{5/4}^2 u^{1/2}\,du \\
&= \frac{2}{3}\,u^{3/2}\Big|_{5/4}^2 \\
&= \frac{2}{3}\left(2^{3/2}-(5/4)^{3/2}\right) \\
&= \frac{4}{3}\sqrt{2}-\frac{5}{12}\sqrt{5} \approx 0.953923
\end{aligned}
$$

9. $\displaystyle\int \frac{x}{1+x^2}\,dx$ Let

$$
\begin{aligned}
u &= 1+x^2 \\
du &= 2x\,dx \\
\frac{1}{2}du &= x\,dx
\end{aligned}
$$

then

$$
\begin{aligned}
\int \frac{x}{1+x^2}\,dx &= \int \frac{1}{2}\cdot\frac{1}{u}\,du \\
&= \frac{1}{2}\int \frac{1}{u}\,du \\
&= \frac{1}{2}\ln u + C \\
&= \frac{1}{2}\ln(1+x^2) + C
\end{aligned}
$$

10. $\displaystyle\int_e^{e^2} \frac{1}{x\ln x}\,dx$ Let

$$
\begin{aligned}
u &= \ln x \\
du &= \frac{1}{x}\,dx
\end{aligned}
$$

then

$$\begin{aligned}
\int_e^{e^2} \frac{1}{x \ln x}\, dx &= \int_1^2 \frac{1}{u}\, du \\
&= \ln u\big|_1^2 \\
&= \ln 2 - \ln 1 \\
&= \ln 2 \approx= 0.693147
\end{aligned}$$