Millersville University
Mathematics Department
MATH 161, *Calculus I*, Derivatives

Examples

Find the first derivatives of the following functions.

1. $f(x) = (x^3 - 7x + 5)\sin x$

$$\begin{aligned} f'(x) &= \left[\frac{d}{dx}(x^3 - 7x + 5)\right]\sin x + (x^3 - 7x + 5)\frac{d}{dx}\sin x \\ &= \left[3x^2 - 7\right]\sin x + (x^3 - 7x + 5)\cos x \\ &= (3x^2 - 7)\sin x + (x^3 - 7x + 5)\cos x \end{aligned}$$

2. $f(x) = \dfrac{\sin x}{x+1}$

$$\begin{aligned} f'(x) &= \frac{\left[\frac{d}{dx}\sin x\right](x+1) - \sin x\frac{d}{dx}(x+1)}{(x+1)^2} \\ &= \frac{[\cos x](x+1) - \sin x(1)}{(x+1)^2} \\ &= \frac{(\cos x)(x+1) - \sin x}{(x+1)^2} \end{aligned}$$

3. $f(x) = (x^2 - 7)^3(x^4 + 1)$

$$\begin{aligned} f'(x) &= \left[\frac{d}{dx}(x^2 - 7)^3\right](x^4 + 1) + (x^2 - 7)^3\frac{d}{dx}(x^4 + 1) \\ &= \left[3(x^2 - 7)^2\frac{d}{dx}(x^2 - 7)\right](x^4 + 1) + (x^2 - 7)^3 4x^3 \\ &= \left[3(x^2 - 7)^2(2x)\right](x^4 + 1) + 4x^3(x^2 - 7)^3 \\ &= 6x(x^2 - 7)^2(x^4 + 1) + 4x^3(x^2 - 7)^3 \end{aligned}$$

4. $f(x) = \dfrac{x+2}{\sqrt{x^2 - 4}}$

$$\begin{aligned} f(x) &= \frac{x+2}{\sqrt{x^2 - 4}} \\ &= \frac{x+2}{(x^2 - 4)^{1/2}} \\ f'(x) &= \frac{\left[\frac{d}{dx}(x+2)\right](x^2 - 4)^{1/2} - (x+2)\frac{d}{dx}(x^2 - 4)^{1/2}}{((x^2 - 4)^{1/2})^2} \end{aligned}$$

$$
\begin{aligned}
&= \frac{[1]\,(x^2-4)^{1/2} - (x+2)(1/2)(x^2-4)^{-1/2}\tfrac{d}{dx}(x^2-4)}{x^2-4} \\
&= \frac{(x^2-4)^{1/2} - (x+2)(1/2)(x^2-4)^{-1/2}(2x)}{x^2-4} \\
&= \frac{(x^2-4)^{1/2} - x(x+2)(x^2-4)^{-1/2}}{x^2-4} \\
&= \frac{\left((x^2-4)^{1/2} - x(x+2)(x^2-4)^{-1/2}\right)}{(x^2-4)}\frac{(x^2-4)^{1/2}}{(x^2-4)^{1/2}} \\
&= \frac{(x^2-4) - x(x+2)}{(x^2-4)^{3/2}} \\
&= -\frac{2x+4}{(x^2-4)^{3/2}}
\end{aligned}
$$

5. $f(x) = \sqrt{x} + \dfrac{1}{2\sqrt{x}} - \dfrac{3}{x\sqrt{x}}$

$$
\begin{aligned}
f(x) &= x^{1/2} + \frac{1}{2}x^{-1/2} - 3x^{-3/2} \\
f'(x) &= \frac{1}{2}x^{-1/2} + \frac{1}{2}\frac{-1}{2}x^{-3/2} - 3\frac{-3}{2}x^{-5/2} \\
&= \frac{1}{2\sqrt{x}} - \frac{1}{4}x^{-3/2} + \frac{9}{2}x^{-5/2} \\
&= \frac{1}{2\sqrt{x}} - \frac{1}{4x^{3/2}} + \frac{9}{2x^{5/2}}
\end{aligned}
$$

6. $f(x) = e^{\sec(2x+3)}$

$$
\begin{aligned}
f'(x) &= e^{\sec(2x+3)}\frac{d}{dx}\sec(2x+3) \\
&= e^{\sec(2x+3)}\sec(2x+3)\tan(2x+3)\frac{d}{dx}(2x+3) \\
&= e^{\sec(2x+3)}\sec(2x+3)\tan(2x+3)2 \\
&= 2\sec(2x+3)\tan(2x+3)e^{\sec(2x+3)}
\end{aligned}
$$

7. $f(x) = x - \ln(1+e^x)$

$$
\begin{aligned}
f'(x) &= 1 - \frac{1}{1+e^x}\frac{d}{dx}(1+e^x) \\
&= 1 - \frac{1}{1+e^x}e^x \\
&= 1 - \frac{e^x}{1+e^x} \\
&= \frac{1+e^x}{1+e^x} - \frac{e^x}{1+e^x}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1+e^x-e^x}{1+e^x} \\
&= \frac{1}{1+e^x}
\end{aligned}
$$

8. $f(x) = \left(e^{1/x}\right)^2$

$$
\begin{aligned}
f(x) &= \left(e^{1/x}\right)^2 \\
&= \left(e^{x^{-1}}\right)^2 \\
&= e^{2x^{-1}} \\
f'(x) &= e^{2x^{-1}} \frac{d}{dx} 2x^{-1} \\
&= e^{2x^{-1}}(-2)x^{-2} \\
&= -\frac{2}{x^2} e^{2x^{-1}} \\
&= -\frac{2}{x^2} e^{2/x}
\end{aligned}
$$

9. $f(x) = (\tan e^x)^3$

$$
\begin{aligned}
f(x) &= 3\left(\tan e^x\right)^2 \frac{d}{dx} \tan e^x \\
&= 3\tan^2 e^x \sec^2 e^x \frac{d}{dx} e^x \\
&= 3\tan^2 e^x \sec^2 e^x (e^x) \\
&= 3e^x \tan^2 e^x \sec^2 e^x
\end{aligned}
$$

10. $f(x) = \dfrac{1+e^{3x}}{\ln(x^2+1)}$

$$
\begin{aligned}
f'(x) &= \frac{\left[\frac{d}{dx}(1+e^{3x})\right]\ln(x^2+1) - (1+e^{3x})\frac{d}{dx}\ln(x^2+1)}{(\ln(x^2+1))^2} \\
&= \frac{[3e^{3x}]\ln(x^2+1) - (1+e^{3x})\frac{1}{x^2+1}\frac{d}{dx}(x^2+1)}{(\ln(x^2+1))^2} \\
&= \frac{3e^{3x}\ln(x^2+1) - (1+e^{3x})\frac{1}{x^2+1}\frac{d}{dx}(x^2+1)}{(\ln(x^2+1))^2} \\
&= \frac{3e^{3x}\ln(x^2+1) - (1+e^{3x})\frac{1}{x^2+1}(2x)}{(\ln(x^2+1))^2} \\
&= \frac{3e^{3x}\ln(x^2+1) - (1+e^{3x})\frac{2x}{x^2+1}}{(\ln(x^2+1))^2} \\
&= \frac{3(x^2+1)e^{3x}\ln(x^2+1) - 2x(1+e^{3x})}{(x^2+1)(\ln(x^2+1))^2}
\end{aligned}
$$