Millersville University
Mathematics Department
MATH 161, *Calculus I*, **Derivatives**

Examples

Find the derivatives of the following functions.

1. $f(x) = 5x^{-3/5}$

$$\begin{aligned} f'(x) &= 5\left(\frac{-3}{5}\right)x^{-3/5-1} \\ &= -3x^{-8/5} \end{aligned}$$

2. $f(x) = \sqrt{x} - \dfrac{1}{\sqrt{x}}$

$$\begin{aligned} f(x) &= \sqrt{x} - \frac{1}{\sqrt{x}} \\ &= x^{1/2} - x^{-1/2} \\ f'(x) &= \frac{1}{2}x^{1/2-1} - \left(\frac{-1}{2}\right)x^{-1/2-1} \\ &= \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2} \\ &= \frac{1}{2x^{1/2}} + \frac{1}{2x^{3/2}} \end{aligned}$$

3. $f(x) = (x^2 + x + 1)(x^2 + 2)$

$$\begin{aligned} f'(x) &= \left[\frac{d}{dx}(x^2 + x + 1)\right](x^2 + 2) + (x^2 + x + 1)\left[\frac{d}{dx}(x^2 + 2)\right] \\ &= [2x + 1](x^2 + 2) + (x^2 + x + 1)[2x] \\ &= 2x^3 + x^2 + 4x + 2 + 2x^3 + 2x^2 + 2x \\ &= 4x^3 + 3x^2 + 6x + 2 \end{aligned}$$

4. $f(x) = \sqrt[3]{x}\left(x + \dfrac{1}{x}\right)$

$$\begin{aligned} f(x) &= \sqrt[3]{x}\left(x + \frac{1}{x}\right) \\ &= x^{1/3}(x + x^{-1}) \\ f'(x) &= \frac{d}{dx}\left[x^{1/3}\right](x + x^{-1}) + x^{1/3}\left[\frac{d}{dx}(x + x^{-1})\right] \\ &= \left[\frac{1}{3}x^{1/3-1}\right](x + x^{-1}) + x^{1/3}\left[1 + (-1)x^{-1-1}\right] \end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{3}x^{-2/3}(x + x^{-1}) + x^{1/3}(1 - x^{-2}) \\
&= \frac{1}{3}(x^{1/3} + x^{-5/3}) + x^{1/3} - x^{-5/3} \\
&= \frac{\sqrt[3]{x}}{3} + \frac{1}{3x^{5/3}} + \sqrt[3]{x} - \frac{1}{x^{5/3}} \\
&= \frac{4\sqrt[3]{x}}{3} - \frac{2}{3x^{5/3}}
\end{aligned}
$$

5. $f(x) = \dfrac{1}{x^4 + x^2 + 1}$

$$
\begin{aligned}
f'(x) &= -\frac{\frac{d}{dx}(x^4 + x^2 + 1)}{(x^4 + x^2 + 1)^2} \\
&= -\frac{4x^3 + 2x}{(x^4 + x^2 + 1)^2}
\end{aligned}
$$

6. $f(x) = \dfrac{x^5}{x^3 - 2}$

$$
\begin{aligned}
f'(x) &= \frac{\left[\frac{d}{dx}x^5\right](x^3 - 2) - x^5\left[\frac{d}{dx}(x^3 - 2)\right]}{(x^3 - 2)^2} \\
&= \frac{5x^4(x^3 - 2) - x^5[3x^2]}{(x^3 - 2)^2} \\
&= \frac{5x^7 - 10x^4 - 3x^7}{(x^3 - 2)^2} \\
&= \frac{2x^7 - 10x^4}{(x^3 - 2)^2}
\end{aligned}
$$

7. $f(x) = \dfrac{1}{1 + \frac{1}{x}}$

$$
\begin{aligned}
f(x) &= \frac{1}{1 + \frac{1}{x}} \\
&= \frac{1}{1 + x^{-1}} \\
f'(x) &= -\frac{\frac{d}{dx}(1 + x^{-1})}{(1 + x^{-1})^2} \\
&= -\frac{-x^{-2}}{(1 + x^{-1})^2} \\
&= \frac{x^{-2})}{(1 + x^{-1})^2} \\
&= \frac{\frac{1}{x^2}}{\left(1 + \frac{1}{x}\right)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\frac{1}{x^2}}{\left(1+\frac{1}{x}\right)^2}\,\frac{x^2}{x^2} \\
&= \frac{1}{(x+1)^2}
\end{aligned}
$$

8. $f(x) = \dfrac{x}{(x^2+2)(x^2+3)}$

$$
\begin{aligned}
f'(x) &= \frac{\left[\frac{d}{dx}x\right](x^2+2)(x^2+3) - x\frac{d}{dx}\left[(x^2+2)(x^2+3)\right]}{\left((x^2+2)(x^2+3)\right)^2} \\
&= \frac{(x^2+2)(x^2+3) - x\left(\left[\frac{d}{dx}(x^2+2)\right](x^2+3) + (x^2+2)\frac{d}{dx}(x^2+3)\right)}{\left((x^2+2)(x^2+3)\right)^2} \\
&= \frac{(x^2+2)(x^2+3) - x\left(2x(x^2+3) + (x^2+2)(2x)\right)}{\left((x^2+2)(x^2+3)\right)^2} \\
&= \frac{x^4+5x^2+6 - x\left(2x^3+6x+2x^3+4x\right)}{\left((x^2+2)(x^2+3)\right)^2} \\
&= \frac{x^4+5x^2+6 - x\left(4x^3+10x\right)}{\left((x^2+2)(x^2+3)\right)^2} \\
&= \frac{x^4+5x^2+6-4x^4-10x^2}{\left((x^2+2)(x^2+3)\right)^2} \\
&= \frac{-3x^4-5x^2+6}{\left((x^2+2)(x^2+3)\right)^2}
\end{aligned}
$$

9. $f(x) = x(x^2+2x+1)(x^3+2)$

$$
\begin{aligned}
f'(x) &= \left[\frac{d}{dx}x\right](x^2+2x+1)(x^3+2) + x\frac{d}{dx}\left[(x^2+2x+1)(x^3+2)\right] \\
&= (x^2+2x+1)(x^3+2) + x\left(\left[\frac{d}{dx}(x^2+2x+1)\right](x^3+2) + (x^2+2x+1)\frac{d}{dx}(x^3+2)\right) \\
&= (x^2+2x+1)(x^3+2) + x((2x+2)(x^3+2) + (x^2+2x+1)3x^2) \\
&= x^5+2x^4+x^3+2x^2+4x+2 + x(2x^4+2x^3+4x+4+3x^4+6x^3+3x^2) \\
&= x^5+2x^4+x^3+2x^2+4x+2+2x^5+2x^4+4x^2+4x+3x^5+6x^4+3x^3 \\
&= 6x^5+10x^4+4x^3+6x^2+8x+2
\end{aligned}
$$

10. $f(x) = \left(\dfrac{x^2+1}{x^2+2}\right)(x^2+3)$

$$
\begin{aligned}
f'(x) &= \left[\frac{d}{dx}\left(\frac{x^2+1}{x^2+2}\right)\right](x^2+3) + \left(\frac{x^2+1}{x^2+2}\right)\frac{d}{dx}(x^2+3) \\
&= \left(\frac{\left[\frac{d}{dx}(x^2+1)\right](x^2+2) - (x^2+1)\frac{d}{dx}(x^2+2)}{(x^2+2)^2}\right)(x^2+3) + \left(\frac{x^2+1}{x^2+2}\right)2x
\end{aligned}
$$

$$
\begin{aligned}
&= \left(\frac{2x(x^2+2)-(x^2+1)(2x)}{(x^2+2)^2}\right)(x^2+3)+\left(\frac{x^2+1}{x^2+2}\right)2x \\
&= \left(\frac{2x^3+4x-(2x^3+2x)}{(x^2+2)^2}\right)(x^2+3)+\left(\frac{2x^3+2x}{x^2+2}\right) \\
&= \left(\frac{2x}{(x^2+2)^2}\right)(x^2+3)+\left(\frac{2x^3+2x}{x^2+2}\right) \\
&= \frac{2x^3+6x}{(x^2+2)^2}+\frac{2x^3+2x}{x^2+2}
\end{aligned}
$$

11. $f(x)=\dfrac{7}{7x^3-4x^2+3x+5}$

$$
\begin{aligned}
f'(x) &= 7\left[\frac{d}{dx}\frac{1}{7x^3-4x^2+3x+5}\right] \\
&= 7\left[\frac{-\frac{d}{dx}(7x^3-4x^2+3x+5)}{(7x^3-4x^2+3x+5)^2}\right] \\
&= 7\left[\frac{-(21x^2-8x+3)}{(7x^3-4x^2+3x+5)^2}\right] \\
&= -\frac{147x^2-56x+21}{(7x^3-4x^2+3x+5)^2}
\end{aligned}
$$