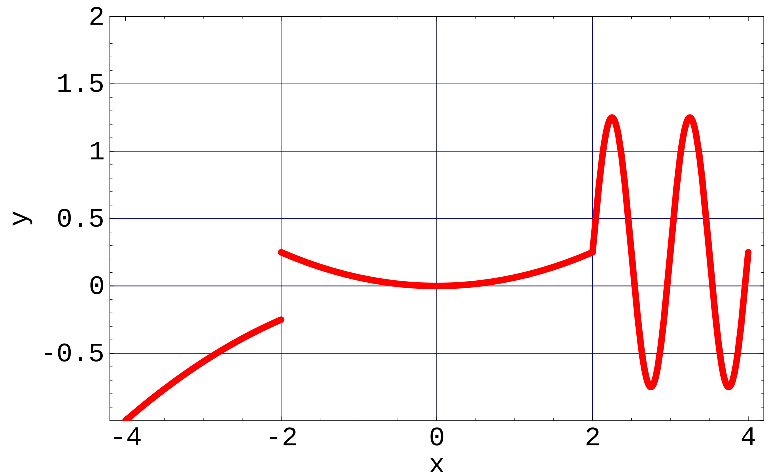
Millersville University
Department of Mathematics
MATH 161, *Calculus I*, Limits

Examples and Answers

1. Use the graph of $f(x)$ show below to find the following limits. If a limit does not exist, be prepared to explain why.



(a) $\displaystyle\lim_{x\to -2^-} f(x) = -\frac{1}{4}$

(b) $\displaystyle\lim_{x\to -2^+} f(x) = \frac{1}{4}$

(c) $\displaystyle\lim_{x\to -2} f(x)$ does not exist. The limit from the left and the limit from the right are different.

(d) $\displaystyle\lim_{x\to 0} f(x) = 0$

(e) $\displaystyle\lim_{x\to 2^+} f(x) = \frac{1}{4}$

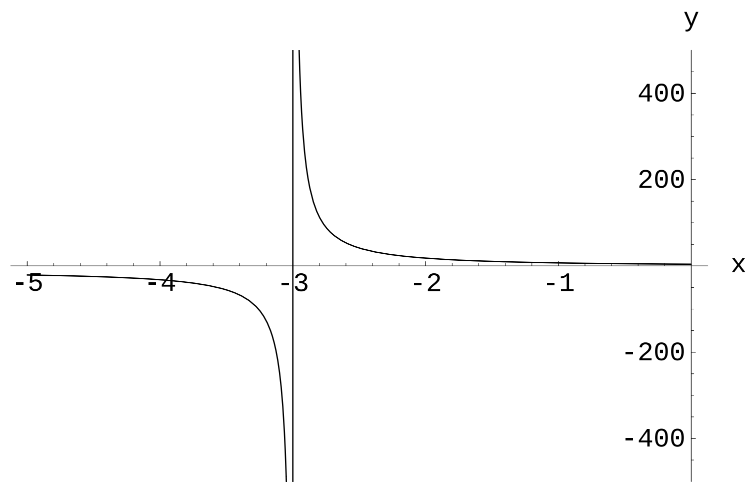
2. Evaluate the following limits, if they exist. If a limit does not exist be prepared to explain why.

(a) $\displaystyle\lim_{x\to 3^+} (x+2)(3x-1) = 40$

$$\begin{aligned}
\lim_{x\to 3^+} (x+2)(3x-1) &= \left(\lim_{x\to 3^+} (x+2)\right)\left(\lim_{x\to 3^+} (3x-1)\right) \\
&= \left(\left[\lim_{x\to 3^+} x\right] + \left[\lim_{x\to 3^+} 2\right]\right)\left(\left[\lim_{x\to 3^+} 3x\right] - \left[\lim_{x\to 3^+} 1\right]\right) \\
&= (3+2)\left(3\left[\lim_{x\to 3^+} x\right] - 1\right)
\end{aligned}$$

$$\begin{aligned} &= 5\left((3)(3)-1\right) \\ &= (5)(8) \\ &= 40 \end{aligned}$$

(b) $\lim_{x \to -3} \dfrac{x^2 - x + 12}{x + 3}$ does not exist. The limit does not exist since the limit of the denominator is 0 and the limit of the numerator is 18. If we look at the graph of $f(x) = \dfrac{x^2 - x + 12}{x+3}$ we can see a vertical asymptote at $x = -3$.



(c) $\lim_{x \to 9^-} \dfrac{9 - x}{3 - \sqrt{x}} = 6$

$$\begin{aligned} \lim_{x \to 9^-} \frac{9-x}{3-\sqrt{x}} &= \lim_{x \to 9^-} \frac{(9-x)(3+\sqrt{x})}{(3-\sqrt{x})(3+\sqrt{x})} \\ &= \lim_{x \to 9^-} \frac{(9-x)(3+\sqrt{x})}{9-x} \\ &= \lim_{x \to 9^-} 3 + \sqrt{x} \\ &= \left(\lim_{x \to 9^-} 3\right) + \left(\lim_{x \to 9^-} \sqrt{x}\right) \\ &= 3 + \sqrt{\lim_{x \to 9^-} x} \\ &= 3 + \sqrt{9} \\ &= 3 + 3 \\ &= 6 \end{aligned}$$

(d) $\lim_{x \to 2} \dfrac{\frac{1}{x} - \frac{1}{2}}{x - 2} = -\dfrac{1}{4}$

$$\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2} = \lim_{x \to 2} \frac{\left(\frac{1}{x} - \frac{1}{2}\right)}{(x-2)} \frac{(2x)}{(2x)}$$

$$
\begin{aligned}
&= \lim_{x\to 2} \frac{\frac{2x}{x} - \frac{2x}{2}}{2x(x-2)} \\
&= \lim_{x\to 2} \frac{2-x}{2x(x-2)} \\
&= \lim_{x\to 2} \frac{-(x-2)}{2x(x-2)} \\
&= \lim_{x\to 2} \frac{-1}{2x} \\
&= \frac{\lim_{x\to 2} -1}{\lim_{x\to 2} 2x} \\
&= \frac{-1}{2\lim_{x\to 2} x} \\
&= \frac{-1}{(2)(2)} \\
&= -\frac{1}{4}
\end{aligned}
$$

(e) $\lim_{x\to 2} \frac{|x-2|}{x-2}$ does not exist. The limit from the left and the limit from the right are different as can be seen in the plot of $f(x) = \frac{|x-2|}{x-2}$ shown below.