Please answer the following questions. Your answers will be evaluated on their correctness, completeness, and use of mathematical concepts we have covered. Please show all work and write out your work neatly. Answers without supporting work will receive no credit.

1. A two-pen corral is to be built. The outline of the corral forms two identical adjoining rectangles. If there is 120 feet of fencing available, what dimensions of the corral will maximize the enclosed area?

The corral is pictured above. The length of the corral is $x$ while its width is $y$. Since there are only 120 feet of fencing available, $x$ and $y$ are related through the equation $2x + 3y = 120$, which is called the constraint equation. The total area of the corral is $A = xy$. If we solve the constraint equation for $y$ we get,

$$y = \frac{120 - 2x}{3} = 40 - \frac{2}{3}x.$$

In the formula for area we can replace $y$ to obtain the area formula,

$$A(x) = x \left( 40 - \frac{2}{3}x \right) = 40x - \frac{2}{3}x^2.$$

The area of the corral will be maximized when $x$ is one of the critical numbers of $A(x)$. Since $0 < x < 60$ we should look for a critical point inside that interval.

$$A'(x) = 40 - \frac{4}{3}x$$

The derivative $A'(x) = 0$ when $x = 30$. Since $A''(x) = -4/3 < 0$ for all $x$ then according to the second derivative test, the area is maximized when $x = 30$. Substituting this value for $x$ into the constraint equation gives $y = 20$. Thus the corral should be 30 feet long and 20 feet wide. It will have a maximum area of 600 square feet.
2. Find the indefinite integral,

\[ \int \left( 5x - \frac{3}{e^x} \right) \, dx. \]

\[ \int \left( 5x - \frac{3}{e^x} \right) \, dx = \int 5x \, dx - \int \frac{3}{e^x} \, dx \]
\[ = 5 \int x \, dx - 3 \int e^{-x} \, dx \]
\[ = 5 \int x \, dx - 3 \left( -e^{-x} \right) \]
\[ = 5 \cdot \frac{1}{2} x^2 + 3e^{-x} \]
\[ = \frac{5}{2} x^2 + 3e^{-x} \]
\[ = \frac{5}{2} x^2 + 3 \cdot \frac{1}{e^x} \]