

Millersville University  
Department of Mathematics  
MATH 161, *Calculus I* Final Exam Review  
April 22, 2004

The following exercises will help you prepare for the final examination. Solutions will be available on Monday, April 26, 2004.

1. Evaluate the following limits if they exist. If a limit does not exist, please explain why.

(a)  $\lim_{x \rightarrow \infty} \frac{\sqrt{2x^2 - x}}{6x - 1}$

(b)  $\lim_{x \rightarrow 3^-} \frac{\sqrt{x} - \sqrt{3}}{x - 3}$

2. Find the value of  $c$  which makes the following function continuous at  $x = -2$ :

$$f(x) = \begin{cases} \frac{x + 10}{c^3} & \text{if } x \leq -2 \\ \frac{c^3}{18} + x + 1 & \text{if } x > -2 \end{cases}$$

3. Find the derivatives of the following functions.

(a)  $f(x) = e^{x^3} + \sin(e^x + x^2) + \ln(2 + \cos x)$

(b)  $g(x) = \left(1 + \frac{1}{x^2} + \frac{1}{x^2 + 1}\right)^2$

(c)  $h(x) = (x^2 - 1)^{15}(x^4 - 3x)$

(d)  $F(x) = \int_0^{\cos x} \sqrt{t^3 + t + 3} dt$

(e)  $G(x) = (x + 1)^{\sin x}$

4. Use the definition of the derivative as the limit of a difference quotient to find the derivative of  $f(x) = \sqrt{x + 1}$ .

5. Use linear approximation to approximate the value of  $e^{1/10}$ .

6. Find the equation of the tangent line to the curve

$$(2x - y)^3 + 6x = 2y + y^2 - 2$$

at the point  $(2, 3)$ .

7. Evaluate the following definite and indefinite integrals.

(a)  $\int_0^1 (x^2 + 2)\sqrt{x^3 + 6x + 5} dx$

(b)  $\int \frac{x + 1}{x + 2} dx$

(c)  $\int \frac{\sec^2 x}{2 + \tan x} dx$

(d)  $\int_0^2 \frac{x^3}{\sqrt{x^4 + 9}} dx$

(e)  $\int_1^4 \frac{x^2 - x + 1}{\sqrt{x}} dx$

$$(f) \int_{-5}^5 3 + \sqrt{25 - x^2} dx$$

8. Suppose that  $f$  is a continuous function such that  $f(3) = -2$  and  $f(7) = 3$ . Show that the equation  $\frac{1}{1+x}f(x) = 0$  for some  $x$  between 3 and 7.
9. A bagel is placed in a  $50^\circ$  cooler. After 6 minutes, the bagel's temperature is  $70^\circ$ ; after 9 minutes, the bagel's temperature is  $60^\circ$ . What was the bagel's initial temperature?
10. A rectangular box with a square bottom and no top is to have a volume of 216 cubic feet. The material for the bottom costs \$8 per square foot, while the material for the sides costs \$4 per square foot. Find the dimensions which give the box with smallest total cost.
11. Graph the function  $f(x) = \frac{e^x}{x-1}$ . Specifically:
  - Find the  $x$ -coordinates of the  $x$ -intercepts and the  $y$ -coordinate of the  $y$ -intercept (if any).
  - Find the intervals on which  $f$  increases and the intervals on which  $f$  decreases.
  - Find the  $x$ -coordinates of any local maxima or minima.
  - Find the intervals on which  $f$  is concave up and the intervals on which  $f$  is concave down.
  - Find the  $x$ -coordinates of any inflection points.
  - Locate any vertical asymptotes, and compute the relevant limits.
  - Locate any horizontal asymptotes, and compute the relevant limits.
  - Make a qualitatively accurate sketch of the graph based on the information above.