Please answer the following questions. Your answers will be evaluated on their correctness, completeness, and use of mathematical concepts we have covered. Please show all work and write out your work neatly. Answers without supporting work will receive no credit. The point values of the problems are listed in parentheses.

1. (10 points each) For the following functions and equations find $\frac{dy}{dx}$. You do not need to fully simplify your answers, but you should write all final answers without negative exponents and without complex fractions (fractions within a numerator or denominator).

   (a) $y = \frac{1 + \tan x}{1 - \sec x}$

   (b) $y = \sqrt{\cos x^2 - \sin x^2}$
(c) \( y = \ln(e^{3x} - e^{-3x}) \)

(d) \( y = e^{(x^2 + 1) \ln x} \)

(e) \( \ln(x + y) + x^2 - 2y^3 = 1 \)
2. (10 points) Using Newton’s Method with an initial approximation of $x_0 = 10$, approximate the solution to the equation

$$5 - x/2 - \cos x = 0.$$ 

Your final answer should be accurate to six decimal places and you should show the values of all the intermediate approximations ($x_1, x_2, etc.$) that you calculate.

3. (10 points) A hot-air balloon is launched vertically from level ground. One hundred feet from the launch point is a pulley which unwinds a rope attached to the balloon. The rope is unwound at a rate of 5 feet per second. At what rate is the balloon rising when 500 feet of rope has been unwound?
4. (10 points) Use a linear approximation to the approximate the value of \( \tan \frac{3 \pi}{4} \).

5. (10 points) Find the absolute maximum and minimum values of the function \( f(x) = 2x^4 - 4x^2 + 3 \) on the interval \([-2, 0]\).
6. (6 points) Use the Mean Value Theorem to explain why the equation

\[-x^3 - ax + 7 = 0\]

has only one real number solution when \(a > 0\). You may assume that the equation has at least one solution, you must show it has only one solution.
7. (1 point each) Match the name of the theorem to the statement of the theorem.

A. Intermediate Value Theorem
B. Rolle’s Theorem
C. Extreme Value Theorem
D. Mean Value Theorem

______ If \( f(x) \) is continuous on the interval \([a, b]\) and differentiable on \((a, b)\), then there exists a number \( a < c < b \) such that
\[
   f'(c) = \frac{f(b) - f(a)}{b - a}.
\]

______ If \( f(x) \) is continuous on the interval \([a, b]\) and differentiable on \((a, b)\) and \( f(a) = f(b) \), then there exists a number \( a < c < b \) such that \( f'(c) = 0 \).

______ If \( f(x) \) is continuous on the interval \([a, b]\), then \( f(x) \) has a global maximum and minimum on \([a, b]\).

______ If \( f(x) \) is continuous on the interval \([a, b]\) and \( k \) lies between \( f(a) \) and \( f(b) \), then there exists a number \( a < c < b \) such that \( f(c) = k \).