Please answer the following questions. Your answers will be evaluated on their correctness, completeness, and use of mathematical concepts we have covered. Please show all work and write out your work neatly. Answers without supporting work will receive no credit. The point values of the problems are listed in parentheses.

1. (10 points each) Evaluate the following indefinite and definite integrals. When appropriate you may use the Fundamental Theorem of Calculus.

   (a) \[ \int_{-1}^{1} (3x^2 - 4x + 7) \, dx \]

   (b) \[ \int \left( 3\sqrt{x} + \frac{4}{x^2} \right) \, dx \]
(c) \[ \int_{-\pi/3}^{\pi} \sec x \tan x \, dx \]

(d) \[ \int \frac{e^{-x}}{e^{-x} + 3} \, dx \]

(e) \[ \int \cos 4x \, dx \]
2. (10 points) Using a Riemann sum with $n = 40$ and left-hand endpoint evaluation, approximate the definite integral

$$\int_{-1}^{2} \frac{1}{2x^2 + 1} \, dx.$$ 

3. (10 points) For the function

$$p(x) = -x^3 + 6x^2 - 9x + 3$$

find the intervals on which the function is increasing and decreasing.
4. A piece of cardboard measures 10 inches by 15 inches. Two equal squares are removed from a 10-inch side as shown in the figure below. Two equal rectangles are removed from the other corners so that the tabs can be folded to form a rectangular box with a lid. Note that the figure below is not drawn to scale.

(a) (3 points) Write a formula $V(x)$ for the volume of the box.

(b) (2 points) What are the largest and smallest values of $x$ that are physically permitted?

(c) (5 points) Find the value of $x$ which maximizes the volume of the box.
5. (10 points) For the function

\[ g(x) = x^3(8 - x) \]

find the intervals on which the function is concave up and concave down. State the \( x \) values for any points of inflection of \( g(x) \).
6. (10 points) The following facts are known about the function \( f(x) \).

- \( f(1) = 0 \)
- \( f'(1) = 0 \)
- \( \lim_{x \to 2^+} f(x) = \infty, \lim_{x \to 2^-} f(x) = -\infty, \) and \( \lim_{x \to 0} f(x) = -\infty \)
- \( \lim_{x \to \infty} f(x) = 0, \lim_{x \to -\infty} f(x) = \infty \)
- \( f''(x) > 0 \) for \( x > 2 \) and \( f''(x) < 0 \) for \( x < 2 \) with \( x \neq 0 \)

Sketch a plausible graph for \( f(x) \) on the axes below.