1. (4 points each) Find the exact values of the following limits if they exist. If a limit does not exist, please explain why.

(a) \( \lim_{x \to -3} \frac{x^2 - x - 12}{x + 3} \)

(b) \( \lim_{x \to 1^+} \frac{1 - 2x}{x^2 - 1} \)

(c) \( \lim_{x \to \infty} \frac{3x^2 - 5x + 4}{2 + 6x - 5x^2} \)
2. (4 points each) Find the following derivatives.

(a) \( \frac{d}{dx} (\sin e^x - 5x^2) \)

(b) \( \frac{d}{dx} \left( \ln x^2 - \frac{3}{x^4} + \tan^2 x \right) \)

(c) \( \frac{d}{dx} (e^x \cos^4 x) \)
3. (4 points) Use a linear approximation to estimate $\sqrt{25.1}$.

4. (6 points) The graph of the equation $xy = \sin y$ is a curve $C$ passing through the point $P = (0, \pi)$. Find the equation of the line tangent to $C$ at point $P$. 
5. (4 points) Use the definition of the derivative as the limit of a difference quotient to find the derivative of \( f(x) = 3x^2 + 2x \). You may not use any shortcut differentiation rules in this problem.

6. (4 points each) Evaluate the following definite and indefinite integrals. You must give an exact answer, not a decimal approximation.

(a) \( \int \frac{x^2 + 1}{x} \, dx \)

(b) \( \int \frac{2x}{1 + x^2} \, dx \)
(c) \[ \int_1^e \frac{\ln x}{4x} \, dx \]

(d) \[ \int_0^{\sqrt{\pi}} x \left(1 + \sin x^2\right) \, dx \]

(e) \[ \int_0^1 \frac{5e^x}{\sqrt{1 + e^x}} \, dx \]
7. (2 points) The graph of function $f(x)$ is shown below. Use the graph to find the value of

$$\int_{-1}^{3} f(x) \, dx.$$ 

8. (5 points) A rectangle in the $xy$-plane has its two lower corners on the $x$-axis and its two upper corners on the graph of $y = 16 - x^2$. For all such rectangles, what are the dimensions of the one with the largest area?
9. (6 points) As hot water fills a teapot, you note that the temperature (at time $t = 0$) is $y(0) = 210^\circ$F. Five minutes later (at time $t = 5$) the tea has cooled to $y(5) = 140^\circ$F. The room temperature is $70^\circ$F. According to Newton’s Law of Cooling, the instantaneous rate of cooling is directly proportional to the difference between the temperature of the tea $y(t)$ and the room temperature. Thus

$$\frac{dy}{dt} = k(y - 70), \quad \text{where } k \text{ is a constant.}$$

(a) Find the temperature function $y(t)$.

(b) What is the temperature of the tea after 10 minutes?

(c) At what time will the temperature of the tea be $87.5^\circ$F?
10. (1 point each) Evaluate the following expressions.

(a) \( \frac{d}{dx} \int_{1}^{x} \frac{1}{2 + \sin t} \, dt \)

(b) \( \int_{\pi}^{2\pi} \frac{d}{dx} \left( \frac{1}{2 + \sin x} \right) \, dx \)

(c) \( \frac{d}{dx} \int_{\pi}^{2\pi} \frac{1}{2 + \sin t} \, dt \)

(d) \( \frac{d}{dx} \int_{1}^{x^2} \frac{1}{2 + \sin t} \, dt \)
11. For the function \( f(x) = 4e^{-x^2/4} - 1 \), the first and second derivatives are
\[
f'(x) = -2xe^{-x^2/4} \quad \text{and} \quad f''(x) = (x^2 - 2)e^{-x^2/4}.
\]

(a) (1 point) Find the domain of \( f \).

(b) (1 point) Find the \( x \)-intercepts (if none, say so).

(c) (1 point) Find the \( y \)-intercept (if none, say so).

(d) (2 points) Find the intervals on which \( f \) is increasing and the intervals on which \( f \) is decreasing.

(e) (2 points) Find the \( x \)-coordinates of any local extrema.

(f) (2 points) Find the intervals on which \( f \) is concave up and the intervals on which \( f \) is concave down.
(g) (2 points) Find the $x$ coordinate(s) of any inflection points (if none, say so).

(h) (2 points) Find any vertical or horizontal asymptotes (if none, say so).

(i) (2 points) Sketch the graph of $f$ based on the information you found in parts (a)-(h).
12. (1 point each) Indicate whether each of the following statements is always True or sometimes False.

(a) The function \( f(x) = \cos x \), for \(-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\), is one-to-one.

(b) \( \frac{d}{dx} 2^x = x2^{x-1} \)

(c) \( \ln \frac{1}{3} = -\int_1^3 \frac{1}{x} \, dx \)

(d) If \( f \) is continuous and \( 1 \leq f(x) \leq 4 \) for all \( x \), then \( 2 \leq \int_{-1}^{1} f(x) \, dx \leq 8 \).

(e) The function \( f(x) = \begin{cases} x & \text{if } x \leq 1 \\ cx^2 & \text{if } x > 1 \end{cases} \) is continuous at \( x = 1 \) if \( c = 1 \).

(f) There exists a function \( f \) such that \( f(x) > 0 \) for all \( x \) and \( \lim_{x \to 1} f(x) = 0 \).

(g) If \( f(1) = 1 \) and \( f(2) = 3 \), then there exists a value \( c \) between 1 and 2 such that \( f(c) = 2 \).

(h) \( \frac{d^2 y}{dx^2} = \left( \frac{dy}{dx} \right)^2 \)

(i) For every function \( f \) which is continuous on \([-1, 2]\), there exists a value \( c \) in \((-1, 2)\) for which \( 3f(c) = f(2) - f(-1) \).

(j) \( \int_{-1}^{1} \frac{1}{x^4} \, dx = -\frac{2}{3} \)

Extra Credit (5 points — No partial credit) Find a continuous function \( f \) such that \( \int_0^x f(t) \, dt = 3f(x) - 2 \) for all \( x \).