Many of your higher level mathematics courses will focus on mathematical conjecture and proof.

Today we will examine a mathematical phenomenon in a specific example and conjecture the general behavior in all cases.

We will then attempt to justify why the behavior is present in the more general case.
Specific Example

1. Select a two-digit number.
2. Reverse the digits.
3. Subtract the smaller from the larger.
4. Reverse the digits in the difference.
5. Add this result to the difference.
Select a two-digit number.
Reverse the digits.
Subtract the smaller from the larger.
Reverse the digits in the difference.
Add this result to the difference.
Specific Example

1. Select a two-digit number.
2. Reverse the digits.
3. Subtract the smaller from the larger.
4. Reverse the digits in the difference.
5. Add this result to the difference.

1 75
2 57
Specific Example

1. Select a two-digit number.
2. Reverse the digits.
3. Subtract the smaller from the larger.
4. Reverse the digits in the difference.
5. Add this result to the difference.

1. 75
2. 57
3. \( 75 - 57 = 18 \)
Specific Example

1. Select a two-digit number.
2. Reverse the digits.
3. Subtract the smaller from the larger.
4. Reverse the digits in the difference.
5. Add this result to the difference.

1 75
2 57
3 $75 - 57 = 18$
4 81
Specific Example

1. Select a two-digit number.
2. Reverse the digits.
3. Subtract the smaller from the larger.
4. Reverse the digits in the difference.
5. Add this result to the difference.

1. 75
2. 57
3. $75 - 57 = 18$
4. 81
5. $81 + 18 = 99$
Repeat this process for the numbers in the table below.

<table>
<thead>
<tr>
<th>Case</th>
<th>XY</th>
<th>Case</th>
<th>XY</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>52</td>
<td>(k)</td>
<td>84</td>
</tr>
<tr>
<td>(b)</td>
<td>21</td>
<td>(l)</td>
<td>69</td>
</tr>
<tr>
<td>(c)</td>
<td>89</td>
<td>(m)</td>
<td>14</td>
</tr>
<tr>
<td>(d)</td>
<td>06</td>
<td>(n)</td>
<td>45</td>
</tr>
<tr>
<td>(e)</td>
<td>65</td>
<td>(o)</td>
<td>32</td>
</tr>
<tr>
<td>(f)</td>
<td>28</td>
<td>(p)</td>
<td>72</td>
</tr>
<tr>
<td>(g)</td>
<td>53</td>
<td>(q)</td>
<td>80</td>
</tr>
<tr>
<td>(h)</td>
<td>40</td>
<td>(r)</td>
<td>46</td>
</tr>
<tr>
<td>(i)</td>
<td>31</td>
<td>(s)</td>
<td>56</td>
</tr>
<tr>
<td>(j)</td>
<td>55</td>
<td>(t)</td>
<td>66</td>
</tr>
</tbody>
</table>
Question: why do we get the results we do?
Three-digit Example

1. Select a three-digit number.
2. Reverse the digits.
3. Subtract the smaller from the larger.
4. Reverse the digits in the difference.
5. Add this result to the difference.
Select a three-digit number.
Reverse the digits.
Subtract the smaller from the larger.
Reverse the digits in the difference.
Add this result to the difference.
Three-digit Example

1. Select a three-digit number.  
2. Reverse the digits.  
3. Subtract the smaller from the larger.  
4. Reverse the digits in the difference.  
5. Add this result to the difference.

1. 751  
2. 157
Three-digit Example

1. Select a three-digit number.
2. Reverse the digits.
3. Subtract the smaller from the larger.
4. Reverse the digits in the difference.
5. Add this result to the difference.

1. 751
2. 157
3. $751 - 157 = 594$
Three-digit Example

1. Select a three-digit number.
2. Reverse the digits.
3. Subtract the smaller from the larger.
4. Reverse the digits in the difference.
5. Add this result to the difference.

1. 751
2. 157
3. $751 - 157 = 594$
4. 495
Three-digit Example

1. Select a three-digit number.
2. Reverse the digits.
3. Subtract the smaller from the larger.
4. Reverse the digits in the difference.
5. Add this result to the difference.

1. 751
2. 157
3. $751 - 157 = 594$
4. 495
5. $495 + 594 = 1089$
Repeat this process for the numbers in the table below.

<table>
<thead>
<tr>
<th>Case</th>
<th>XYZ</th>
<th>Case</th>
<th>XYZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>124</td>
<td>(k)</td>
<td>997</td>
</tr>
<tr>
<td>(b)</td>
<td>055</td>
<td>(l)</td>
<td>692</td>
</tr>
<tr>
<td>(c)</td>
<td>230</td>
<td>(m)</td>
<td>031</td>
</tr>
<tr>
<td>(d)</td>
<td>536</td>
<td>(n)</td>
<td>171</td>
</tr>
<tr>
<td>(e)</td>
<td>048</td>
<td>(o)</td>
<td>848</td>
</tr>
<tr>
<td>(f)</td>
<td>243</td>
<td>(p)</td>
<td>412</td>
</tr>
<tr>
<td>(g)</td>
<td>173</td>
<td>(q)</td>
<td>139</td>
</tr>
<tr>
<td>(h)</td>
<td>265</td>
<td>(r)</td>
<td>939</td>
</tr>
<tr>
<td>(i)</td>
<td>499</td>
<td>(s)</td>
<td>510</td>
</tr>
<tr>
<td>(j)</td>
<td>779</td>
<td>(t)</td>
<td>339</td>
</tr>
</tbody>
</table>
**Question:** why do we get the results we do?
The digits of a base eight number come from the set 
\{0, 1, 2, 3, 4, 5, 6, 7\}

Each octal position in a base eight number is associated 
with a power of 8.
The digits of a base eight number come from the set 
\{0, 1, 2, 3, 4, 5, 6, 7\}
Each octal position in a base eight number is associated with a power of 8.

**Example**

\[160_8 = 1(8^2) + 6(8^1) + 0(8^0) = 64 + 48 + 0 = 112_{10}\]
Three-digit Base-8 Example

1. Select a 3-digit number in base 8.
2. Reverse the digits.
3. Subtract the smaller from the larger.
4. Reverse the digits in the difference.
5. Add this result to the difference.
Select a 3-digit number in base 8.

Reverse the digits.

Subtract the smaller from the larger.

Reverse the digits in the difference.

Add this result to the difference.
Select a 3-digit number in base 8.
Reverse the digits.
Subtract the smaller from the larger.
Reverse the digits in the difference.
Add this result to the difference.
Three-digit Base-8 Example

1. Select a 3-digit number in base 8.
2. Reverse the digits.
3. Subtract the smaller from the larger.
4. Reverse the digits in the difference.
5. Add this result to the difference.

1. $751_8$
2. $157_8$
3. $751_8 - 157_8 = 572_8$
Three-digit Base-8 Example

1. Select a 3-digit number in base 8.
2. Reverse the digits.
3. Subtract the smaller from the larger.
4. Reverse the digits in the difference.
5. Add this result to the difference.

1. \(751_8\)
2. \(157_8\)
3. \(751_8 - 157_8 = 572_8\)
4. \(275_8\)
Select a 3-digit number in base 8.
Reverse the digits.
Subtract the smaller from the larger.
Reverse the digits in the difference.
Add this result to the difference.
Repeat this process for the base-8 numbers in the table below.

<table>
<thead>
<tr>
<th>Case</th>
<th>XYZ</th>
<th>Case</th>
<th>XYZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>772</td>
<td>(k)</td>
<td>402</td>
</tr>
<tr>
<td>(b)</td>
<td>314</td>
<td>(l)</td>
<td>013</td>
</tr>
<tr>
<td>(c)</td>
<td>356</td>
<td>(m)</td>
<td>561</td>
</tr>
<tr>
<td>(d)</td>
<td>043</td>
<td>(n)</td>
<td>606</td>
</tr>
<tr>
<td>(e)</td>
<td>145</td>
<td>(o)</td>
<td>623</td>
</tr>
<tr>
<td>(f)</td>
<td>763</td>
<td>(p)</td>
<td>462</td>
</tr>
<tr>
<td>(g)</td>
<td>036</td>
<td>(q)</td>
<td>535</td>
</tr>
<tr>
<td>(h)</td>
<td>572</td>
<td>(r)</td>
<td>270</td>
</tr>
<tr>
<td>(i)</td>
<td>003</td>
<td>(s)</td>
<td>160</td>
</tr>
<tr>
<td>(j)</td>
<td>556</td>
<td>(t)</td>
<td>037</td>
</tr>
</tbody>
</table>
Question: why do we get the results in base 8 that we do?
**Question:** what results should we get if we perform this process in bases 2, 3, 4, 5, 6, 7, 9?
Question: what results should we get if we perform this process in bases 2, 3, 4, 5, 6, 7, 9?

Followup question: can we verify (prove) this conjecture?
The digits of a hexadecimal number come from the set \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}.

For example

\[ CAFE_{16} = 12(16^3) + 10(16^2) + 15(16^1) + 14 = 51966_{10}. \]

- Perform the familiar process on several 3-digit hexadecimal numbers and conjecture the general results.
- Justify (prove) your conjecture.