Page 433, Exercise 22c

Find the volume of the solid of revolution generated by revolving the region bounded between \( y = x + 2 \), \( y = -x - 2 \), and \( x = 0 \) around the \( y \)-axis.

The region to be revolved is shown in the graph below.

If we use the Method of Shells only one integral must be evaluated. The Method of Disks would require two integrals. The generic definition integral of the Method of Shells can be thought of as

\[
V = 2\pi \int_a^b \text{(radius of shell)} \text{(height of shell)} \, dx.
\]

In this case the radius of a shell will be the horizontal distance from the \( y \)-axis to a cross section of a shell. This can be expressed as radius = 0 - \( x = -x \). The height of a shell will be the vertical distance from the top boundary of the region to the bottom boundary of the region. This can be expressed as height = \( (x + 2) - (-x - 2) = 2x + 4 \). Thus the volume is

\[
V = 2\pi \int_{-2}^{0} (-x)(2x + 4) \, dx = 2\pi \int_{-2}^{0} 2x^2 + 4x \, dx
\]

\[
= -2\pi \left[ \frac{2}{3}x^3 + 2x^2 \right]_{-2}^{0}
\]

\[
= 2\pi \left( \frac{2}{3}(-2)^3 + 2(-2)^2 \right) = 2\pi \left( \frac{-16}{3} + 8 \right) = \frac{16\pi}{3}
\]
Page 440, Exercise 14

Numerically determine the arc length of the graph of $y = x^{1/3}$ for $1 \leq x \leq 2$. By definition the arc length of the curve will be

$$L = \int_{1}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$= \int_{1}^{2} \sqrt{1 + \left(\frac{1}{3}x^{-2/3}\right)^2} \, dx$$

$$= \int_{1}^{2} \sqrt{1 + \frac{1}{9}x^{-4/3}} \, dx$$

$$\approx 1.03377$$