Page 604, Exercise 28

Find the indicated limit.
\[ \lim_{x \to 0} \frac{\sec x - 1}{x^2} \]

This limit is indeterminate of the form 0/0 since
\[ \lim_{x \to 0} \sec x - 1 = 0 \quad \text{and} \quad \lim_{x \to 0} x^2 = 0. \]

Applying l’Hôpital’s Rule yields
\[ \lim_{x \to 0} \frac{\sec x - 1}{x^2} = \lim_{x \to 0} \frac{\sec x \tan x}{2x} \quad \text{indeterminate 0/0} \]
\[ = \lim_{x \to 0} \frac{\sec x \tan^2 x + \sec^3 x}{2} \]
\[ = \frac{1}{2}. \]

Page 617, Exercise 30

Determine whether the integral converges or diverges. If it converges, find the value of the integral.
\[ \int_{-\infty}^{\infty} \frac{1}{\sqrt{x}} \, dx \]

This integral is improper because both the upper and lower limits of integration are infinite and because the integrand has a discontinuity at \( x = 0 \). We can rewrite the integral in the form
\[ \int_{-\infty}^{\infty} \frac{1}{\sqrt{x}} \, dx = \int_{-\infty}^{-1} \frac{1}{\sqrt{x}} \, dx + \int_{-1}^{0} \frac{1}{\sqrt{x}} \, dx + \int_{0}^{1} \frac{1}{\sqrt{x}} \, dx + \int_{1}^{\infty} \frac{1}{\sqrt{x}} \, dx \]

We can say the original integral converges provided that all four improper integrals on the right-hand side of the equation converge. If one of them diverges, then the original integral diverges.

Since \( \frac{1}{2} \leq \frac{1}{\sqrt{x}} \) for \( x \geq 1 \) and since
\[ \int_{1}^{\infty} \frac{1}{x} \, dx \]
diverges, then by the Comparison Test,
\[ \int_{1}^{\infty} \frac{1}{\sqrt{x}} \, dx \]
diverges as well. Therefore
\[ \int_{-\infty}^{\infty} \frac{1}{\sqrt{x}} \, dx \quad \text{diverges.} \]