Page 634, Exercise 24

Determine whether the sequence converges or diverges.

\[ a_n = (-1)^n \frac{4}{n+1} \]

Since

\[ -\frac{4}{n+1} \leq a_n \leq \frac{4}{n+1} \]

and

\[ \lim_{n \to \infty} -\frac{4}{n+1} = 0 \quad \text{and} \quad \lim_{n \to \infty} \frac{4}{n+1} = 0 \]

then by the Squeeze Theorem for Sequences

\[ a_n = (-1)^n \frac{4}{n+1} \to 0 \quad \text{as} \quad n \to \infty. \]

Page 645, Exercise 26

Determine if the series converges or diverges. If the series converges, find its sum.

\[ \sum_{k=0}^{\infty} (-1)^{k+1} \frac{4}{3^k} \]

We note that

\[ \sum_{k=0}^{\infty} (-1)^{k+1} \frac{4}{3^k} = \sum_{k=0}^{\infty} (-4) \frac{(-1)^k}{3^k} = \sum_{k=0}^{\infty} (-4) \left( \frac{-1}{3} \right)^k. \]

If we assign \( a = -4 \) and \( r = -\frac{1}{3} \), then the original series is an example of a geometric series. Since \( |r| = \frac{1}{3} < 1 \) then the series converges to

\[ \sum_{k=0}^{\infty} (-1)^{k+1} \frac{4}{3^k} = \frac{-4}{1 - (-\frac{1}{3})} = \frac{-4}{\frac{4}{3}} = -3. \]