Page 738, Exercise 32

Find the area enclosed by the curve given below.

\[ \begin{align*}
  x &= t \sin t, \\
  y &= t \cos t, \\
  &\quad \frac{-\pi}{2} \leq t \leq \frac{\pi}{2}
\end{align*} \]

The graph of the plane curve is shown below. The orientation of the curve is clockwise.

The area inside the curve is

\[
A = \int_{-\pi/2}^{\pi/2} y(t)x'(t) \, dt \\
= \int_{-\pi/2}^{\pi/2} (t \cos t)(\sin t + t \cos t) \, dt \\
= \int_{-\pi/2}^{\pi/2} t \cos t \sin t + t^2 \cos^2 t \, dt \\
= \int_{-\pi/2}^{\pi/2} \frac{t \sin 2t}{2} + \frac{t^2}{2}(1 + \cos 2t) \, dt \\
= \frac{1}{2} \int_{-\pi/2}^{\pi/2} t \sin 2t + t^2 + t^2 \cos 2t \, dt \\
= \frac{1}{2} \int_{-\pi/2}^{\pi/2} t \sin 2t \, dt + \frac{1}{2} \int_{-\pi/2}^{\pi/2} t^2 \, dt + \frac{1}{2} \int_{-\pi/2}^{\pi/2} t^2 \cos 2t \, dt \\
= -\frac{1}{4} t \cos 2t + \frac{1}{8} \sin 2t + \frac{1}{6} t^3 + \frac{1}{4} t^2 \sin 2t + \frac{1}{4} t \cos 2t - \frac{1}{8} \sin 2t \bigg|_{-\pi/2}^{\pi/2} \\
= \frac{\pi^3}{24} \approx 1.29193
\]
Page 745, Exercise 18

Find the arc length of the following curve.

\[ \begin{align*}
  x &= \sin t \\
  y &= \sin \sqrt{2}t , \quad 0 \leq t \leq \pi
\end{align*} \]

The arc length of the curve is given by the definite integral,

\[ L = \int_0^\pi \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} \, dt \]

\[ = \int_0^\pi \sqrt{\cos^2 t + 2 \cos^2 \sqrt{2}t} \, dt \]

\[ \approx 3.72343 \]