1. (6 points each) Determine whether the following sequences converge or diverge. If a sequence converges, find the exact value of the limit.

(a) \( \left\{ \frac{n}{1 + \sqrt{n}} \right\} \)

(b) \( \left\{ \frac{\ln(2 + e^n)}{3n} \right\} \)

(c) \( \left\{ \arctan \left( \frac{2n}{2n+1} \right) \right\} \)
2. (7 points each) Determine whether each of the following integrals is convergent or divergent. If an integral is convergent, find its exact value. Not all of the following integrals are improper integrals.

(a) \[ \int_{0}^{\infty} \frac{x}{1+3x^2} \, dx \]

(b) \[ \int_{2}^{3} \frac{1}{x^3 + x^2 - 2x} \, dx \]
3. (7 points each) Determine whether the series is convergent or divergent. You do not need to find the values of convergent series. You must justify your answers.

(a) \( \sum_{n=1}^{\infty} \frac{3n^2}{2n^2 + 1} \)

(c) \( \int_{0}^{1} \frac{1}{\sqrt{x}} \, dx \)
(b) \( \sum_{n=1}^{\infty} \frac{\arctan n}{1 + n^2} \)

(c) \( \sum_{n=1}^{\infty} \frac{\sin^2 n}{n \sqrt{n}} \)
4. (7 points each) Find the exact values of the following limits.

(a) \( \lim_{x \to 0} \frac{\cos(mx) - \cos(nx)}{x^2} \), \( m \) and \( n \) are constants.
(b) \( \lim_{x \to 0} (\cos 3x)^{5/x} \)

(c) \( \lim_{x \to \infty} \frac{\ln x}{x^p} \), where \( p > 0 \) is constant.
5. (5 points) Find the values of $x$ for which the following series converges. Find the sum of the series for those values of $x$.

$$
\sum_{n=1}^{\infty} \frac{x^n}{3^n}
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