Please answer the following questions. Your answers will be evaluated on their correctness, completeness, and use of mathematical concepts we have covered. Please show all work and write out your work neatly. Answers without supporting work will receive no credit. The point values of the problems are given in parentheses.

1. (5 points each) Evaluate the following indefinite and definite integrals.

   (a) \( \int x^3 \ln x \, dx \)
(b) \[ \int \frac{x^2 + 3x + 6}{x^3 - 4x} \, dx \]
(c) \[ \int \frac{1}{x^2 \sqrt{1 + x^2}} \, dx \]

(d) \[ \int_{0}^{\pi/4} \tan^2 x \sec^4 x \, dx \]
(e) \[ \int_{0}^{\pi/4} \frac{\cos x}{\sqrt{\sin x}} \, dx \]

2. (5 points each) Consider the curve described by the parametric equations:

\[ \begin{align*}
    x & = \frac{3t}{1 + t^3} \\
    y & = \frac{3t^2}{1 + t^3}
\end{align*} \]

(a) Find the points on the curve where the tangent lines are horizontal.
(b) Assuming that the $x$-$y$ equation for the curve is $x^3 + y^3 = 3xy$, find a polar coordinate equation for the curve.

(c) Find the area enclosed by the loop in the graph of this curve. You may approximate the area numerically.
3. (6 points) Find the arc length of the graph of the polar coordinate curve,

\[ r = 4 + 3 \sin \theta. \]

You may approximate the arc length numerically.

4. (5 points each) Determine if the following series are absolutely convergent, conditionally convergent, or divergent. You must justify your answers.

(a) \[ \sum_{k=1}^{\infty} (-1)^{k-1} k^{-1/3} \]
(b) \[ \sum_{k=1}^{\infty} \frac{(-1)^k (k + 1) 3^k}{2^{2k+1}} \]

(c) \[ \sum_{k=2}^{\infty} \frac{(-1)^k \sqrt{k}}{\ln k} \]
(d) \[ \sum_{k=1}^{\infty} \frac{\cos\left(\frac{k\pi}{6}\right)}{k\sqrt{k}} \]

5. (6 points) Find the radius and interval of convergence of the series,

\[ \sum_{k=1}^{\infty} \frac{(-2)^k}{\sqrt{k}} (x + 1)^k. \]
6. (6 points each) Find the exact value of the following limits, if they exist.

(a) \( \lim_{x \to 0} (1 - 2x)^{1/x} \)

(b) \( \lim_{x \to \infty} \left( \frac{1}{x} - \frac{1}{e^x - 1} \right) \)
7. (6 points) Find a Taylor series for $f(x) = x^3$ centered at $c = -1$.

8. (5 points) The curve described by the equation $y = 1 - x^2$ with $0 \leq x \leq 1$ is revolved around the $y$-axis. Find the volume of the resulting solid of revolution.
9. (5 points) A gate in a dam is in the shape of a semicircle whose diameter is at the bottom of the dam (see figure below). The diameter of the gate is 4 feet. The bottom of the gate is 10 feet below the surface of the water. Find the hydrostatic force on the gate. You may approximate the hydrostatic force numerically.