Please answer the following question. You may be asked to show your work on this sheet. Please write your work legibly and show all steps in your work. If your work is legible and correct, your answer will be marked correct. The point value for the problem on this sheet is 10 points.

Given the equation $y = x^2 - 4x + 3$, find the x-intercepts and the vertex of the parabola.

1. Vertex: $(x, y) = (2, -1)$
2. x-intercepts: $x = 1$ and $x = 3$
(a) Given the curve given in parametric form:
\[ \begin{align*}
  x &= t^3 + 1 \\
  y &= t^2 + 1
\end{align*} \]

Find the slope of the tangent line to the curve as a function of \( t \):
\[ \frac{dy}{dx} = \frac{2t}{3t^2} \]

(b) Find the point on the curve where the tangent line is horizontal. (If none:
\[ \begin{align*}
  &\frac{dy}{dx} = 0 \quad \text{so} \quad t = 0 \\
  &\left( f(t) \right)_{t=0} = (0,-1)
\end{align*} \]

(c) Find the point on the curve where the tangent line is vertical. (If none:
\[ \begin{align*}
  &\frac{dx}{dt} = 0 \quad \text{so} \quad t = \pm \sqrt{2} \\
  &\left( f(t) \right)_{t=\pm \sqrt{2}} = (1,3), (1,3)
\end{align*} \]
5. (10 points) The curve is given by the parametric form:

\[ x = a \cos t, \quad y = b \sin t, \quad 0 \leq t \leq \pi \]

where \( a \) and \( b \) are positive constants. Find the arc length of the curve.

The arc length \( L \) is given by the integral:

\[ L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt \]

For the parametric form above:

\[ \frac{dx}{dt} = -a \sin t \]
\[ \frac{dy}{dt} = b \cos t \]

Thus, the integral becomes:

\[ L = \int_{0}^{\pi} \sqrt{(-a \sin t)^2 + (b \cos t)^2} \, dt \]

\[ L = \int_{0}^{\pi} \sqrt{a^2 \sin^2 t + b^2 \cos^2 t} \, dt \]

\[ L = \int_{0}^{\pi} \sqrt{a^2 \sin^2 t + b^2 \cos^2 t} \, dt \]

\[ L = \int_{0}^{\pi} \sqrt{a^2 (1 - \cos^2 t) + b^2 \cos^2 t} \, dt \]

\[ L = \int_{0}^{\pi} \sqrt{a^2 + b^2 - b^2 \cos^2 t} \, dt \]

\[ L = \int_{0}^{\pi} \sqrt{a^2 + b^2} \, dt \]

\[ L = \sqrt{a^2 + b^2} \left[ t \right]_{0}^{\pi} \]

\[ L = \pi \sqrt{a^2 + b^2} \]

8. (20 points) For the curve, find the area enclosed by the curve and the x-axis.

The area \( A \) is given by the integral:

\[ A = \int_{a}^{b} y \, dx \]

For the curve \( x = a \cos t, y = b \sin t \):

\[ A = \int_{0}^{\pi} b \sin t \, d(a \cos t) \]

\[ A = \int_{0}^{\pi} b \sin t \, (-a \sin t) \, dt \]

\[ A = -ab \int_{0}^{\pi} \sin^2 t \, dt \]

\[ A = -ab \left[ \frac{t}{2} - \frac{\sin 2t}{4} \right]_{0}^{\pi} \]

\[ A = -ab \left( \frac{\pi}{2} - 0 \right) \]

\[ A = -ab \frac{\pi}{2} \]

\[ A = \frac{ab \pi}{2} \]

A: 6
(1) (cont.) Use some of the linear equations (29) to find linear coefficients and solve for $x$. Then $x = f(t)$.

$\begin{align*}
\omega & = \frac{1}{2} (2) \\
\frac{1}{2} & = \frac{1}{\omega} \\
\frac{1}{\omega^2} & = 2.
\end{align*}$
(10 points) Find all polar equations for the Cartesian equation $y = x^2$.

\[ r = \frac{2a}{1 - \cos \theta} \]

(10 points) Find the area of the region bounded by the polar curve $r = 1 - \cos \theta$.

\[ A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 d\theta \]

where $r = 1 - \cos \theta$, $\alpha = 0$, and $\beta = \pi$.