Please answer the following questions. **Answers without justifying work will receive no credit.** Partial credit will be given as appropriate, do not leave any problem blank.

1. (12 points) Find the position function of a particle whose acceleration function is

   \[ a(t) = (e^{2t}, -t, \sin t), \]

   whose initial velocity vector is \( v(0) = (4, -2, 4) \), and whose initial position vector is \( r(0) = (0, 4, -2) \).

2. (12 points) Find the value of the following limit.

   \[
   \lim_{(x,y) \to (0,0)} \frac{x^2y + xy^3}{x^2 + y^2}
   \]
3. (2 points each) Match the following four functions to their surface plots.

<table>
<thead>
<tr>
<th>Function</th>
<th>Surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x, y) = e^{-x^2} + e^{-4y^2} )</td>
<td></td>
</tr>
<tr>
<td>( f(x, y) = \sin \sqrt{x^2 + y^2} )</td>
<td></td>
</tr>
<tr>
<td>( f(x, y) = y^4 - 8y^2 - 4x^2 )</td>
<td></td>
</tr>
<tr>
<td>( f(x, y) = \frac{x^2y^2 e^{-x^2-y^2}}{x^2 + y^2} )</td>
<td></td>
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</tbody>
</table>
4. (12 points) Find the directional derivative of \( f(x, y) = y^3 \ln x \) in the direction of vector \( \mathbf{v} = (-1, 4) \).

5. (12 points) Show that the following limit does not exist.

\[
\lim_{(x,y)\to(0,0)} \frac{2x^3 y}{4x^4 + 3y^4}
\]
6. (12 points) If \( f(x, y) = \ln \sqrt{x^2 + y^2} \), then show that

\[
\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0
\]
7. (4 points each) If \( z = x \ln y \) and \( x = r + st \) and \( y = 3rst \) find the following partial derivatives.

(a) \( \frac{\partial z}{\partial r} \)

(b) \( \frac{\partial z}{\partial s} \)

(c) \( \frac{\partial z}{\partial t} \)
8. (2 points each) Determine whether each of the following statements is always true or sometimes false.

(a) \( f_{xy} = \frac{\partial^2 f}{\partial y \partial x} \).

(b) If \( f \) is a function then \( \lim_{(x,y) \to (2,5)} f(x, y) = f(2,5) \).

(c) \( D_j f(x, y) = f_x(x, y) \).

(d) If \( f(x, y) \to L \) as \( (x, y) \to (a, b) \) along a straight line through \( (a, b) \), then \( \lim_{(x,y) \to (a,b)} f(x, y) = L \).

(e) If \( f(x, y) = \ln y \), then \( f_x(x, y) = 1/y \).
9. (5 points) Find the equation of tangent plane to the surface \( f(x, y) = xe^{xy^2} + 3y^2 \) where \((x, y) = (1, 1)\).
10. (5 points) Consider the sphere with equation

\[ x^2 + y^2 + z^2 = R^2 \]

where \( R > 0 \) is a constant. Find the equation of the line perpendicular to the surface of the sphere passing through the point \((x_0, y_0, z_0)\) on the surface of the sphere (hint: this means \( x_0^2 + y_0^2 + z_0^2 = R^2 \)).