Please answer the following questions. **Answers without justifying work will receive no credit.** Partial credit will be given as appropriate, do not leave any problem blank.

1. (9 points) Find the following limit if it exists.

\[
\lim_{t \to 1} \left( \frac{e^t - 1}{t} i + \frac{\sqrt{t + 1} - 1}{t} j + \frac{3}{t + 1} k \right)
\]

2. (8 points) Find the unit tangent vector for \( \mathbf{r}(t) = ti + (2 \sin t)j + (3 \cos t)k \) at \( t = \pi/6 \).
3. (10 points) Find the tangential and normal components of acceleration for \( \mathbf{r}(t) = (1, t, t^2) \).

4. (9 points) Evaluate the following definite integral.

\[
\int_1^4 \left( \sqrt{t} \mathbf{i} + te^{-t} \mathbf{j} + \frac{1}{t^2} \mathbf{k} \right) \, dt
\]
5. (2 points each) Indicate whether the statement is always **True** or sometimes **False** for vector-valued functions or functions of several variables.

________(a) The derivative of a vector-valued function is obtained by differentiating each component function.

________(b) For differentiable functions \( u(t) \) and \( v(t) \),
\[
\frac{d}{dt} (u(t) \times v(t)) = u'(t) \times v'(t).
\]

________(c) For a differentiable function \( u(t) \),
\[
\frac{d}{dt} \|u(t)\| = \|u'(t)\|.
\]

________(d) If \( T(t) \) is the unit tangent vector of a smooth curve, then the curvature is \( \kappa = \frac{dT}{dt} \).

________(e) The binormal vector is \( B(t) = T(t) \times N(t) \).

________(f) \( f_{xy} = \frac{\partial^2 f}{\partial x \partial y} \)

________(g) If \( f(x, y) \to L \) as \( (x, y) \to (a, b) \) along every straight line through \( (a, b) \), then
\[
\lim_{(x,y) \to (a,b)} f(x, y) = L.
\]
6. (10 points) Find the curvature of \( \mathbf{r}(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{j} + (\sin t)\mathbf{k} \).

7. (10 points) A football is kicked toward the goal post from a distance of 35 yards. The ball has an initial speed of 75 feet per second and leaves the ground at an angle of 30° above the horizontal. To count as a field goal the football must pass over the cross bar of the goal post at a height of at least 10 feet. Will this kick score a field goal?
8. (2 points each) Match the following four functions to their contour plots.

<table>
<thead>
<tr>
<th>Function</th>
<th>Contour</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x, y) = xy$</td>
<td>(a)</td>
</tr>
<tr>
<td>$f(x, y) = x^2 - y^2$</td>
<td>(c)</td>
</tr>
<tr>
<td>$f(x, y) = y - \cos x$</td>
<td>(b)</td>
</tr>
<tr>
<td>$f(x, y) = e^{1+x^2+y^2}$</td>
<td>(d)</td>
</tr>
</tbody>
</table>
9. (5 points each) Find the following limits if they exist or show that they do not exist.

(a) \[ \lim_{(x,y) \to (0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1} \]

(b) \[ \lim_{(x,y,z) \to (0,0,0)} \frac{xy + yz + xz}{x^2 + y^2 + z^2} \]
10. (4 points each) For the function \( f(x, y, z) = x^5 + x^4y^3z^3 + yz^2 + \cos(xy) \), find the indicated partial derivatives.

(a) \( f_x \)

(b) \( f_{yx} \)

(c) \( f_{xyz} \)