Please answer the following questions. **Answers without justifying work will receive no credit.** Partial credit will be given as appropriate, do not leave any problem blank.

1. (10 points each) Evaluate the following double integrals.

   (a) \( \int \int_\mathcal{R} ye^x \, dA \), where \( \mathcal{R} \) is the triangular region with vertices \((0, 0)\), \((2, 4)\), and \((0, 4)\).

   (b) \( \int \int_\mathcal{R} e^{-x^2-y^2} \, dA \), where \( \mathcal{R} \) is the region in the first quadrant that lies between the circles \( x^2 + y^2 = 9 \) and \( x^2 + y^2 = 16 \).
2. (12 points) Find the local minimum and local maximum values and the saddle points (if any) of the following function:

\[ f(x, y) = x^2 + y^2 + x^2y + 4 \]
3. (5 points each) If $f(u, v, w) = uv + uw + vw$ and $u = xy$, $v = x + y$, and $w = x^2$ then find

(a) $\frac{\partial f}{\partial x}$

(b) $\frac{\partial f}{\partial y}$
4. (5 points each) Consider the function $f(x, y) = \ln(x^2 + y^2)$ and the point with coordinates $(x_0, y_0) = (1, 2)$.

(a) Find the gradient of $f(x, y)$.

(b) Find the direction of the minimum rate of change of $f(x, y)$ at $(x_0, y_0)$.

(c) Find the magnitude of the maximum rate of change of $f(x, y)$ at $(x_0, y_0)$.

(d) Find the directional derivative of $f(x, y)$ at $(x_0, y_0)$ in the direction of $u = \langle 1, -2 \rangle$. 
5. (11 points) Find the equation of the tangent plane to the surface defined by the function

\[ f(x, y) = 9x^2 + y^2 + 6x - 3y + 5 \]

at \((x, y) = (1, 2)\).
6. (12 points) Use Lagrange Multipliers to find the maximum and minimum values of \( f(x, y) = x^2 + y^2 \) subject to the constraint, \( x^4 + y^4 = 1 \).
7. (15 points) A lamina occupies the region in the first quadrant bounded by $y = x^2$ and $y = 1$. The density of the lamina is given by the function $\rho(x, y) = xy$. Find the mass, moments about the $x$ and $y$-axes, and the center of mass of the lamina.