8. (20 marks) Determine the following vector field is conservative. If it is, find its potential function.

\[ \mathbf{F}(x,y) = (x + y) \mathbf{i} + (x - y) \mathbf{j} \]

8. (15 marks) Find the volume of the region above \( z = x^2 + y^2 \) and below \( z = 2 - x^2 - y^2 \).

\[
\iiint_D z \, dV = \int_0^2 \int_{-\sqrt{2-z}}^{\sqrt{2-z}} \int_{-\sqrt{2-z-x^2}}^{\sqrt{2-z-x^2}} dz \, dx \, dy
\]

\[
= \int_0^2 \left[ \frac{1}{2} \left( 2 - x^2 \right) \right] \sqrt{2-z} \, dx
\]

\[
= \frac{1}{2} \int_0^2 (2 - x^2) \sqrt{2-z} \, dx
\]

\[
= \frac{1}{2} \left[ \frac{2}{3} (2-z)^{3/2} \right]_0^2
\]

\[
= \frac{1}{2} \left[ \frac{2}{3} (2-2) - \frac{2}{3} (2-0) \right]
\]

\[
= \frac{1}{3} \cdot \frac{2}{2} = \frac{1}{3}
\]
5. Evaluate the line integral
\[ \int_C x^2 \, dx \\
\text{where } C \text{ is the parabola } x = y^2 \text{ from } (0, 0) \text{ to } (1, 1). \]

\[ x = \sqrt{2} \sqrt{x} + y^2 \sqrt{2} \]
\[ y = \sqrt{2} \sqrt{x} + y^2 \sqrt{2} \]
\[ z = \sqrt{2} (x + y) \]
\[ z = \sqrt{2} (x + y) \]

6. Evaluate the surface integral
\[ \iint_D f(x, y, z) \, dS \]
\[ \text{where } D \text{ is bounded by } z = \sqrt{2} \sqrt{x} + y^2 \text{ and the xy-plane.} \]
8. (10 points) The Green's Function to obtain the following integral.

\[ f(x) = \int \frac{g(a)}{b - x} \, da \]

where \( g(a) \) is the function to be integrated.

\[
\begin{array}{c|c|c}
\text{a} & \text{b} & \text{f(a)} \\
\hline
1 & 2 & 3.0 \\
3 & 4 & 5.0 \\
6 & 7 & 7.0 \\
9 & 10 & 8.0 \\
12 & 15 & 9.0 \\
15 & 20 & 10.0 \\
\end{array}
\]

\[ \int_{a=1}^{15} \frac{g(a)}{b-a} \, da = 90.0 \]

9. (10 points) The potential energy field is

\[ V(x, y) = \frac{1}{x^2 + y^2} \]

where \( x \) and \( y \) are the coordinates in the potential field.

Find the potential energy of the point at \( (x, y) = (2, 3) \).

\[ V(2, 3) = \frac{1}{2^2 + 3^2} = \frac{1}{13} \]

\[ V(2, 3) = \frac{1}{\sqrt{13}} \]

The potential energy at \( (2, 3) \) is \( \frac{1}{\sqrt{13}} \).