1. (6 points) Use the divergence theorem to calculate the flux of
\[ \mathbf{F}(x, y, z) = 3y^2z^3i + 9x^2yz^2j - 4xy^2k \]
across the surface \( S \) of the cube with vertices \((\pm 1, \pm 1, \pm 1)\).
2. (6 points) Evaluate the integral

\[ \iint_{R} (x^2 + y^2) \, dA \]

where \( R \) is the region bounded by the spirals \( r = \theta \) and \( r = 2\theta \) for \( 0 \leq \theta \leq 2\pi \).
3. (7 points) Use Green’s theorem to find the work done by the force

$$\mathbf{F}(x, y) = x(x + y)i + xy^2j$$

in moving a particle from the origin along the $x$-axis to $(1, 0)$, then along the line segment to $(0, 1)$, and then back to the origin along the $y$-axis.
4. (6 points) Determine whether or not $F(x, y) = (1 + 2xy + \ln x)i + x^2j$ is a conservative vector field. If it is, find a function $f$ such that $\nabla f = F$. 
5. (8 points) Find the curl and the divergence of

\[ \mathbf{F}(x, y, z) = xe^{yz} \mathbf{i} + ye^{xz} \mathbf{j} + ze^{xy} \mathbf{k}. \]

Please simplify your results.
6. (8 points) Find the extrema and saddle points (if any) of

\[ f(x, y) = x^3 y + 12x^2 - 8y. \]
7. (8 points) If \( w = \sqrt{x} + y^2 / z \), where \( x = e^{2t} \), \( y = t^3 + 4t \), and \( z = t^2 - 4 \), find \( \frac{dw}{dt} \). Please simplify your results.
8. (6 points) Find the following limit, if it exists, or show that the limit does not exist.

$$\lim_{(x,y,z) \to (0,0,0)} \frac{xy + yz + xz}{x^2 + y^2 + z^2}$$
9. (6 points) Suppose \( \mathbf{a} = 5\mathbf{i} - 6\mathbf{j} - \mathbf{k} \) and \( \mathbf{b} = 3\mathbf{i} + \mathbf{k} \) and find a unit vector perpendicular to both \( \mathbf{a} \) and \( \mathbf{b} \).
10. (6 points) Use cylindrical coordinates to evaluate

$$\iiint_Q \sqrt{x^2 + y^2} \, dV$$

where $Q$ is the region that lies inside the cylinder $x^2 + y^2 = 4$ and between the planes $z = -5$ and $z = 4$. 
11. (6 points) Show that every plane that is tangent to the cone $z^2 = x^2 + y^2$ passes through the origin.
12. (6 points) Identify the surface whose equation in spherical coordinates is

\[ \rho^2 (\sin^2 \phi - 4 \cos^2 \phi) = 1. \]

13. (6 points) A particle moves with position function

\[ \mathbf{r}(t) = 2\sqrt{2} \mathbf{i} + e^{2t} \mathbf{j} + e^{-2t} \mathbf{k}. \]

Find the velocity and acceleration of the particle.
14. (3 points each) Suppose that over a certain region of space the electrical potential is given by
\[ V(x, y, z) = 5x^2 - 3xy + xyz. \]

(a) Find the rate of change of the electrical potential at the point with coordinates \((3, 4, 5)\) in the direction of the vector \(\mathbf{a} = (1, 1, -1)\).

(b) In which direction does \(V\) change most rapidly at the point \((3, 4, 5)\)?

(c) What is the maximum rate of change of \(V\) at the point \((3, 4, 5)\)?
15. (6 points) Let $H$ be the upper half of the sphere $x^2 + y^2 + z^2 = 4$ and let $P$ be the portion of the paraboloid $z = 4 - x^2 - y^2$ above the $xy$-plane. If $F$ is a vector field whose components have continuous partial derivatives explain why

$$\int\int_H (\nabla \times F) \cdot \mathbf{n} dS = \int\int_P (\nabla \times F) \cdot \mathbf{n} dS.$$