

Millersville University  
Department of Mathematics

Name \_\_\_\_\_

MATH 365, *Ordinary Differential Equations*, Homework 07  
October 20, 2008

Find the solutions to the following exercises. Answers without justifying work will receive no credit. Partial credit will be given as appropriate, do not leave any problem blank. Each problem is worth 10 points. Your completed assignment is due at class time on Friday, October 24, 2008.

1. When a mass  $m$  is at the end of a vertically hanging spring, the period is 1.5 seconds. After adding 3 kilograms to the mass, the period becomes 2.5 seconds. What was the original mass  $m$ ?

2. If a hole were bored through the earth's center, one would find that an object dropped in the hole is acted upon by a force of attraction varying directly as the distance between the object and the earth's center. Assuming the earth is a sphere with a radius of 4000 miles, find the period of motion of the object dropped in the hole and the velocity of the object as it passes through the earth's center.

3. Suppose that the position of a simple harmonic oscillator is described by the function  $u(t) = Re^{-\alpha t} \cos(\omega t - \delta)$ . Find the times at which the position is a local extremum.

4. Suppose that the position of a simple harmonic oscillator is described by the function  $u(t) = Re^{-\alpha t} \cos(\omega t - \delta)$ . Suppose  $u(t_n)$  and  $u(t_{n+1})$  are two successive local extreme displacements from equilibrium. Find the ratio  $\frac{u(t_{n+1})}{u(t_n)}$ .

5. Suppose the motion of a forced, undamped simple harmonic oscillator is described by the IVP

$$\begin{aligned}mu'' + ku &= \begin{cases} t & \text{if } 0 \leq t \leq T \\ 0 & \text{if } t \geq T \end{cases} \\u(0) &= 0 \\u'(0) &= 0.\end{aligned}$$

Find the solution to the IVP.

6. A mass on a spring undergoes a forced vibration given by

$$mu'' + ku = F_0 \cos^3 \omega t.$$

Show that there are two values of  $\omega$  at which resonance occurs and find them.