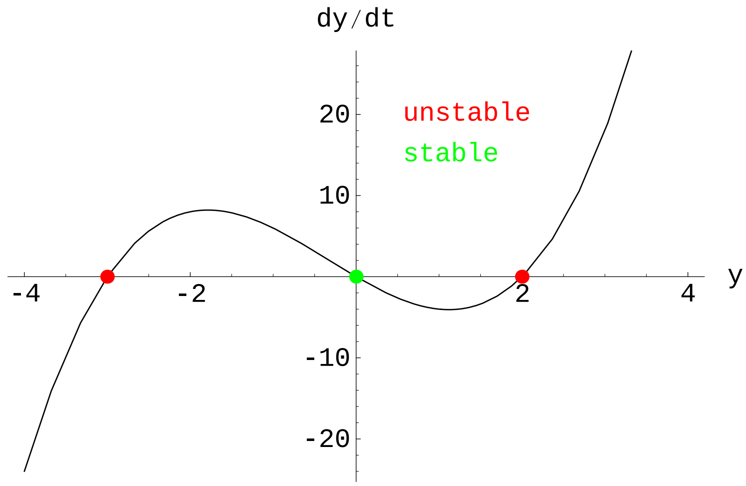
Millersville University
Department of Mathematics
MATH 365, Ordinary Differential Equations, Homework 3
January 30, 2004

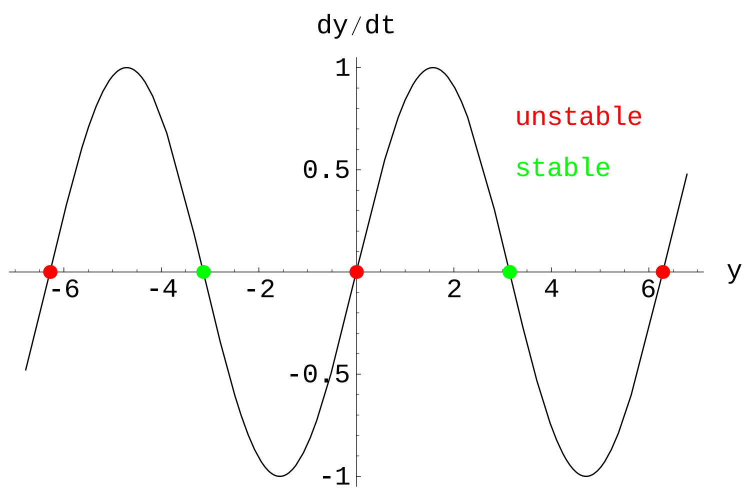
Please answer the following questions. Answers without justifying work will receive no credit. Partial credit will be given as appropriate, do not leave any problem blank. Each problem is worth 10 points. Your completed assignment is due at class time on Wednesday, February 4, 2004.

1. Find the equilibria for the following ordinary differential equations. For each equilibrium point found, state whether the point is stable or unstable.

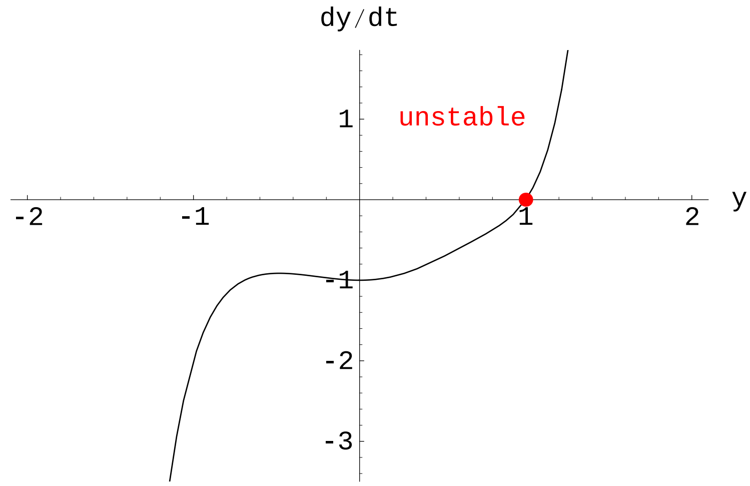
   (a) $\frac{dy}{dt} = y(y-2)(y+3)$ As can be seen in the graph below, the equilibria are $y = -3$ (unstable), $y = 0$ (stable), and $y = 2$ (unstable).



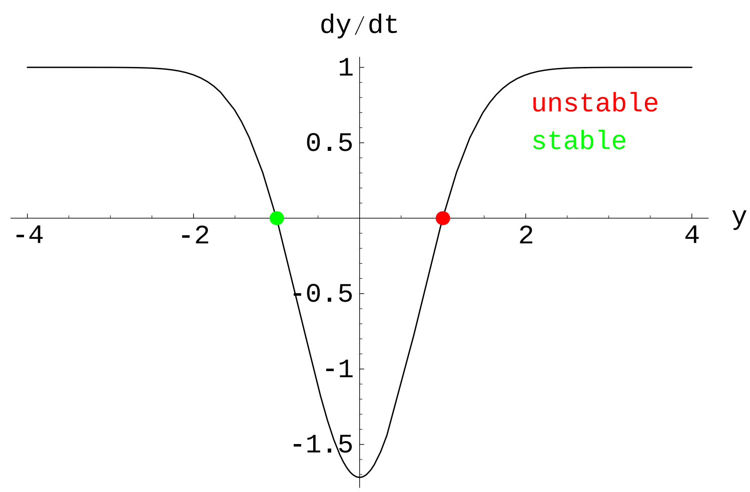
   (b) $\frac{dy}{dt} = \sin y$ As can be seen in the graph below, the equilibria are multiples of π. Even multiples of π (*i.e.* numbers of the form $y = 2n\pi$ where n is an integer) are unstable, while odd multiples of π (*i.e.* numbers of the form $y = (2n+1)\pi$ where n is an integer) are stable.



   (c) $\frac{dy}{dt} = (y^3 - 1)(y^4 - y^2 + 1)$ As can be seen in the graph below, there is a single unstable equilibrium at $y = 1$.

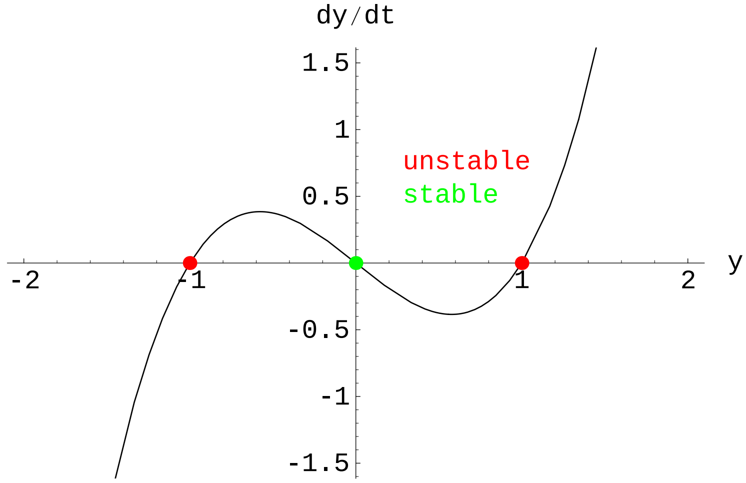


(d) $\frac{dy}{dt} = 1 - e^{1-y^2}$ As can be seen in the graph below, the equilibria are $y = -1$ (stable) and $y = 1$ (unstable).



2. Create a differential equation for which solutions starting between -1 and 1 asymptotically limit on 0 as $t \to \infty$, solutions starting below -1 approach $-\infty$ as $t \to \infty$, and solutions starting above 1 limit on positive infinity.

   One solution among many is $dy/dt = y(y-1)(1+y) = y^3 - y$. As can be seen in the graph below, the equilibria are $y = -1$ (unstable), $y = 0$ (stable), and $y = 1$ (unstable).


dy/dt
1.5
1
0.5
-0.5
-1
-1.5
-2
-1
1
2
y
unstable
stable