Millersville University **Name** **Answer Key**
Department of Mathematics
MATH 365, *Ordinary Differential Equations*, Homework 01
January 14, 2009

Please answer the following questions. Answers without justifying work will receive no credit. Partial credit will be given as appropriate, do not leave any problem blank. Each problem is worth 10 points. Your completed assignment is due at class time on Friday, January 16, 2009.

1. Please complete the following table. In each case y is the dependent variable and t is the independent variable. All other letters represent constants.

Differential Equation	**Order**	**Linear ? (y/n)**	**Autonomous ? (y/n)**
$y'' = -ky$	2	yes	yes
$ty'' + y' + ty = 0$	2	yes	no
$y + ty' = y^2$	1	no	no
$E\,I\,y^{(4)} = f(t)$	4	yes	no
$y'' + 4y' + 5y = 100\sin 20t$	2	yes	no
$y'' = \frac{w}{H}\sqrt{1+(y')^2}$	2	no	yes

2. Determine if $y(t) = e^{-t} + t - 1$ is a solution to the ordinary differential equation

$$y' + y = t.$$

Since $y'(t) = -e^{-t} + 1$ if we substitute $y(t)$ and $y'(t)$ into the given ODE we obtain

$$\begin{aligned} -e^{-t} + 1 + e^{-t} + t - 1 &= t \\ t &= t \end{aligned}$$

and the ODE is satisfied. Thus $y(t)$ is a solution to the ODE.

3. Determine if $y(t) = c_1 e^{-2t} + c_2 e^{t} + c_3 e^{3t}$ is a solution to the ordinary differential equation

$$y''' - 2y'' - 5y' + 6y = 0.$$

The expressions c_i for $i = 1, 2, 3$ are constants.

If we calculate the first three derivatives of $y(t)$ we have

$$\begin{aligned} y'(t) &= -2c_1 e^{-2t} + c_2 e^{t} + 3c_3 e^{3t} \\ y''(t) &= 4c_1 e^{-2t} + c_2 e^{t} + 9c_3 e^{3t} \\ y'''(t) &= -8c_1 e^{-2t} + c_2 e^{t} + 27c_3 e^{3t}. \end{aligned}$$

Substituting these derivatives and $y(t)$ into the ODE yields

$$\begin{aligned} 0 &= -8c_1 e^{-2t} + c_2 e^{t} + 27c_3 e^{3t} - 2(4c_1 e^{-2t} + c_2 e^{t} + 9c_3 e^{3t}) \\ &\quad - 5(-2c_1 e^{-2t} + c_2 e^{t} + 3c_3 e^{3t}) + 6(c_1 e^{-2t} + c_2 e^{t} + c_3 e^{3t}) \\ &= (-8 - 8 + 10 + 6)c_1 e^{-2t} + (1 - 2 - 5 + 6)c_2 e^{t} + (27 - 18 - 15 + 6)c_3 e^{3t} \\ &= 0 \end{aligned}$$

and the ODE is satisfied. Thus $y(t)$ is a solution to the ODE.

4. Determine if $y(t) = At^3 + Bt^{-4} - \frac{1}{3}t^2$ is a solution to the ordinary differential equation

$$t^2 y'' + 2ty' - 12y = 2t^2.$$

The symbols A and B are constants.

If we calculate the first two derivatives of $y(t)$ we obtain

$$\begin{aligned} y'(t) &= 3At^2 - 4Bt^{-5} - \frac{2}{3}t \\ y''(t) &= 6At + 20Bt^{-6} - \frac{2}{3}. \end{aligned}$$

Substituting these derivatives and $y(t)$ into the ODE yields

$$\begin{aligned} 2t^2 &= t^2\left(6At + 20Bt^{-6} - \frac{2}{3}\right) + 2t\left(3At^2 - 4Bt^{-5} - \frac{2}{3}t\right) \\ &\quad - 12\left(At^3 + Bt^{-4} - \frac{1}{3}t^2\right) \\ &= (6 + 6 - 12)At^3 + (20 - 8 - 12)Bt^{-4} + \left(-\frac{2}{3} - \frac{4}{3} + \frac{12}{3}\right)t^2 \\ &= 2t^2 \end{aligned}$$

and the ODE is satisfied. Thus $y(t)$ is a solution to the ODE.

5. A change of variable can sometimes enable us to switch the roles of the independent and dependent variables in an ordinary differential equation. Starting with the equation

$$\frac{dy}{dt} = \frac{1}{\frac{dt}{dy}}$$

differentiate both sides with respect to t to show that

$$\frac{d^2y}{dt^2} = -\frac{\frac{d^2t}{dy^2}}{\left(\frac{dt}{dy}\right)^3}.$$

$$\begin{aligned} \frac{dy}{dt} &= \frac{1}{\frac{dt}{dy}} \\ \frac{d}{dt}\left[\frac{dy}{dt}\right] &= \frac{d}{dt}\left[\frac{1}{\frac{dt}{dy}}\right] \end{aligned}$$

$$
\begin{aligned}
\frac{d^2y}{dt^2} &= \frac{-\frac{d}{dt}\left[\frac{dt}{dy}\right]}{\left(\frac{dt}{dy}\right)^2} \\
&= \frac{-\frac{d}{dy}\left[\frac{dt}{dy}\right]\frac{dy}{dt}}{\left(\frac{dt}{dy}\right)^2} \\
&= \frac{-\frac{d}{dy}\left[\frac{dt}{dy}\right]}{\left(\frac{dt}{dy}\right)^3} \\
&= -\frac{\frac{d^2t}{dy^2}}{\left(\frac{dt}{dy}\right)^3}
\end{aligned}
$$

6. Use the identity established in exercise 5 to re-write the following ordinary differential equation as one with t as the independent variable.

$$
\frac{d^2t}{dy^2} + (\sin t)\left(\frac{dt}{dy}\right)^3 = 0
$$

$$
\begin{aligned}
\frac{d^2t}{dy^2} + (\sin t)\left(\frac{dt}{dy}\right)^3 &= 0 \\
-\left(\frac{dt}{dy}\right)^3\left(\frac{d^2y}{dt^2}\right) + (\sin t)\left(\frac{dt}{dy}\right)^3 &= 0 \\
-\frac{d^2y}{dt^2} + \sin t &= 0 \\
\frac{d^2y}{dt^2} - \sin t &= 0
\end{aligned}
$$