General First Order Equations

Recall the form of a general first-order ODE:

\[ \frac{dy}{dt} = f(t, y) \]

This form is equivalent to the following:

\[ \frac{dy}{dt} - f(t, y) = 0 \]

Suppose this (possibly non-linear) form can be algebraically manipulated into the following form:

\[ M(t) \, dt + N(y) \, dy = 0 \]

This type of ordinary differential equation is called **separable**.

Examples

Verify that the following first-order ODEs are separable.

\[ \frac{dy}{dt} = \frac{t^2}{y \left(1 + t^3\right)} \]

\[ \frac{dy}{dt} = \frac{y^2}{t} \]

\[ \frac{dy}{dt} = \frac{3 \, t^2}{3 \, y^2 - 4} \]

\[ \frac{dy}{dt} = \frac{at + b}{ct + d} \]
Solving a Separable ODE

Given a separable ODE:

\[ M(t) \, dt + N(y) \, dy = 0 \]

we may re-write it as:

\[ N(y) \, dy = -M(t) \, dt \]

and integrate both sides of the equation.

Example (1 of 3)

\[ \frac{dy}{dt} = \frac{t^2}{y \left(1 + t^3\right)} \]

```
In[1]:= ContourPlot[3 y^2 - 2 Log[Abs[1 + t^3]], {t, -1, 2}, {y, -1, 1},
            FrameLabel -> {"t", "y"}, Contours -> 10, ContourShading -> False]
```

Example (2 of 3)

\[ \frac{dy}{dt} = \frac{y^2}{t} \]
Find the solution to the ODE given above which satisfies the initial condition: \( y(1)=2 \).

\[
\text{In[3]:=} \quad \text{ContourPlot}[\log(t) + \frac{1}{y}, \{t, 0.01, 2\}, \{y, 0.01, 2\}, \\
\quad \text{FrameLabel} \rightarrow \{"t", "y"\}, \text{Contours} \rightarrow 10, \text{ContourShading} \rightarrow \text{False}]
\]
Find the solution to the ODE given above which satisfies the initial condition: \( y(1)=0 \).
\begin{verbatim}
In[5]:= ContourPlot[t^3 - (y^3 - 4 y), {t, -3, 3}, {y, -3, 3},
FrameLabel -> {"t", "y"}, Contours -> {1}, ContourShading -> False]
\end{verbatim}

\begin{center}
\includegraphics[width=\textwidth]{contour_plot.png}
\end{center}

\section*{Homework}

Read Section 2.2

Pages 47–50: work exercises 1, 5, 9, 13, 17, 21, 30, 31