

Millersville University
Mathematics Department

Name _____

MATH 365, *Ordinary Differential Equations*, Final Examination
December 11, 2008, 10:15AM-12:15PM

Please answer the following questions. Show all work and write neatly. Answers without justifying work will receive no credit. Partial credit will be given as appropriate, do not leave any problem blank. The point values of problems are indicated in parentheses.

1. (8 points each) Solve the following ordinary differential equations and initial value problems.

(a) $x\sqrt{1+y^2} dx = y\sqrt{1+x^2} dy; \quad y(0) = 0$

(b) $y'' - 4y = 4e^{2t}$

$$(c) \quad x^2 y'' + xy' - 9y = \sqrt{x} + \frac{1}{\sqrt{x}}, \quad x > 0$$

(d) $(x - y \cos x) dx - (\sin x + y) dy = 0$

(e) $y'' + 3y' + 2y = \delta(t - 2) + u_3(t); \quad y(0) = y'(0) = 0$

The functions δ and u refer to the Dirac delta function and unit step functions respectively.

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2. (6 points) Deer hunting season begins November 1st and lasts for thirty days. Hunters kill 450 deer per month during the season. The deer population grows exponentially at a rate of 0.057/month. If there are 500 deer in an isolated region (where no deer can leave or enter) on January 1st, how many deer will be in the region on the following January 1st?

3. (12 points) Find two linearly independent solutions to the differential equation below **using a power series expansion** about $x_0 = 0$. State the **recurrence relation** and find a formula for the **general term** of each power series.

$$y'' + y = 0$$

4. A 256 lb weight is suspended from a vertical spring having spring constant of 200 lb/ft. The weight is raised 3 inches above its equilibrium position and released.

(a) (6 points) Find the displacement $u(t)$ of the mass for all $t > 0$.

(b) (4 points) Find the period and amplitude of the vibration.

(c) (4 points) Find the position of the mass at a time $t = \pi/3$ after release and determine the direction and speed of the mass.

5. Consider the ordinary differential equation,

$$4xy'' + 2y' + y = 0.$$

(a) (2 points) Show that $x = 0$ is a regular singular point for this equation.

(b) (4 points) Find the exponents of singularity corresponding to the regular singular point $x = 0$.

- (c) (6 points) Find a series solution to the equation corresponding to the larger of the exponents of singularity. Write out at least the first four terms of the series.

6. (8 points) Are the functions e^{3x} and xe^{3x} linearly independent on the real number line? You must justify your answer.

7. (8 points) Using the Convolution Theorem find the inverse Laplace transform of the following function. You must express the result in the form of a convolution integral.

$$Y(s) = \frac{s^2}{s^4 - 1}$$