Millersville University **Name** ______________________________
Department of Mathematics
MATH 365, *Ordinary Differential Equations*, Test 1
September 24, 2008

Please answer the following questions. Show all work and write neatly. Answers without justifying work will receive no credit. Partial credit will be given as appropriate, do not leave any problem blank. The point values of problems are indicated in parentheses.

1. (12 points) The human liver can be treated as a tank with a volume of 500 cm^3. A drug is entering the liver via a blood vessel. Blood flows to the liver at a rate of 10 cm^3 per second. The drug in the incoming blood has a concentration of 0.08 g/cm^3. Blood flows from the liver at a rate of 10 cm^3 per second and the concentration of drug in the outgoing blood equals the well-mixed concentration of drug in the liver. Assuming the liver contained none of the drug initially, find the concentration of the drug in the liver after 120 seconds.

2. (10 points each) Solve each of the following ODEs or IVPs.

(a) $t^2y^2\frac{dy}{dt} = (t^2+1)(y^3-1)$

(b) $t\frac{dy}{dt} + y = t^2, \quad y(1) = 2.$

(c) $\dfrac{dy}{dt} = \dfrac{t}{y} e^{t^2 - y^2}$

(d) $(\sin y)\, dt + (t \cos y - y)\, dy = 0$

3. (8 points) Let $y(t) = c_1 t^{-2} + c_2 t^{-2} \ln t$ and show that y solves the ODE below.

$$t^2 y'' + 5ty' + 4y = 0.$$

4. (8 points each) Suppose the total mass of a boat and its passenger is m, the velocity of the boat is $v(t)$ and quantities P, S, and B are positive constants. Newton's Second Law of Motion models the motion of the boat as

$$m\frac{dv}{dt} = \frac{8P}{v} - BSv^2.$$

(a) Determine all the equilibria for this ODE.

(b) Classify each equilibrium as stable or unstable.

5. (12 points) Show that the following ODE is not exact and find an integrating factor depending on a single variable that makes the equation exact. You do not need to solve the resulting exact equation.

$$(2t + 2ty^2)\,dt + (t^2y + 2y + 3y^3)\,dy = 0$$

6. (12 points) Consider the second order linear constant coefficient homogeneous ODE

$$y'' - 2y' + y = 0.$$

Show that $\{e^t,\, te^t\}$ is a fundamental set of solutions to this ODE.