Millersville University **Name** _____
Mathematics Department
MATH 365, *Ordinary Differential Equations*, Test 2
October 29, 2008, 2:00-2:50PM

Please answer the following questions. Show all work and write neatly. Answers without justifying work will receive no credit. Partial credit will be given as appropriate, do not leave any problem blank. The point values of problems are indicated in parentheses.

1. Consider the second order linear ordinary differential equation,

$$y'' - \frac{2}{t+1}y' + \frac{2}{(t+1)^2}y = 0.$$

(a) (4 points) Verify that $y_1(t) = t + 1$ is a solution to the ODE.

(b) (8 points) Using reduction of order, find a second linearly independent solution to this ODE.

2. (10 points each) Find the general solutions to the following ordinary differential equations.

   (a) $2y'' - 5y' + 2y = 0$

   (b) $y'' - 6y' + 13y = 3e^{-2t}$

(c) $y'' + 2y' + y = 0$

(d) $y'' - 2y' + y = \ln t$

3. Suppose that an object of mass $m = 1$ kg is attached to a spring with spring constant $k = 100$ N/m. The coefficient of damping in the spring/mass system is $\gamma = 6$ N/m/sec. The mass is set into motion from its equilibrium position with an upward velocity of 2 m/sec.

   (a) (6 points) Write down the initial value problem describing the displacement of the mass.

   (b) (10 points) Solve the initial value problem just found.

(c) (4 points) What is the quasi–frequency of the spring/mass system?

(d) (4 points) If the spring/mass system is modified so that there is no damping present and then subjected to an external force of the form $F_0 \cos \omega t$, for what value of ω will resonance occur?

4. (8 points) Determine whether the following pair of functions is linearly independent for $-\infty < t < \infty$.

$$y_1(t) = \sin^2 t, \quad y_2(t) = 1 - \cos 2t$$

5. (8 points) Solve the following ODE by means of a power series solution about $t_0 = 0$. You must state the recurrence relation and the first four terms in each of two linearly independent solutions.

$$(2 + t^2)y'' - ty' + 4y = 0$$

6. (8 points) Show that $t_0 = \pi/2$ is an ordinary point for the following ODE. If $y(t) = \sum_{n=0}^{\infty} a_n \left(t - \frac{\pi}{2}\right)^n$ is a power series solution to the ODE, what is a lower bound for the radius of convergence of the series?

$$(\sin t)y'' - t\,y' + 2y = 0$$