

Millersville University
Mathematics Department

Name _____

MATH 365, *Ordinary Differential Equations*, Test 3
November 24, 2008, 2:00-2:50PM

Please answer the following questions. Show all work and write neatly. Answers without justifying work will receive no credit. Partial credit will be given as appropriate, do not leave any problem blank. The point values of problems are indicated in parentheses.

1. (10 points) The Chebyshev equation is

$$(1 - x^2)y'' - xy' + \alpha^2y = 0$$

where α is a constant. Show that $x = 1$ is a regular singular point and find the exponents of singularity.

2. (15 points each) Solve the following ODEs/IVPs.

(a) $t^2y'' + 3ty' + 5y = 0$

(b) $2t^2y'' - 6ty' + 8y = 0$

(c) $t^2 y'' + 2ty' + \frac{17}{4}y = 0; y(1) = 2, y'(1) = -3$

3. (15 points) Consider the ODE

$$xy'' + 2y = 0$$

which has a regular singular point at $x = 0$. Find the first three nonzero coefficients of a solution to the ODE corresponding to the larger of the two exponents of singularity.

4. (15 points) Use the definition of the Laplace transform to find $\mathcal{L}\{t^2 e^{at}\}$, where a is a constant.

5. (15 points) Use the Laplace transform to solve the following IVP.

$$y'' + 3y' + 2y = 0$$

$$y(0) = 0$$

$$y'(0) = 1$$