

Millersville University
Mathematics Department

Name _____

MATH 365, *Ordinary Differential Equations*, Test 2
March 5, 2004

Please answer the following questions. Show all work and write neatly. Answers without justifying work will receive no credit. Partial credit will be given as appropriate, do not leave any problem blank. The point values of problems are indicated in parentheses.

1. (10 points) Legendre's differential equation is

$$(1 - t^2)y'' - 2ty' + \alpha(\alpha + 1)y = 0,$$

where α is a real number. Without solving Legendre's equation, find the Wronskian of two linearly independent solutions to the equation.

2. Solve the following ordinary differential equations and initial boundary value problems.

(a) (11 points) $y'' + 3y' - 18y = 0$; $y(0) = 1$, $y'(0) = 0$

(b) (15 points) $y'' + 2y = t + 3e^{2t}$

(c) (8 points) $4y'' - 12y' + 9y = 0$

(d) (11 points) $y'' + 2y' + 5y = 0$; $y(0) = 0$, $y'(0) = 1$

(e) (15 points) $y'' - 2y' + y = \frac{e^t}{1+t^2}$

3. (10 points) Are the functions $f(t) = \cos^2 t$ and $g(t) = 1 + \cos 2t$ linearly independent on the set of real numbers?

4. (10 points) Suppose $y(t)$ is any solution to the equation

$$y'' + (3 - \alpha)y' - 2(\alpha - 1)y = 0.$$

Find the value of α for which $\lim_{t \rightarrow \infty} y(t) = 0$.

5. (10 points) If $y_1(t) = \sin(t^2)$ solves the differential equation

$$ty'' - y' + 4t^3y = 0,$$

then find a second linearly independent solution to the equation.