Millersville University **Name** _____
Mathematics Department
MATH 365, *Ordinary Differential Equations*, **Test 3**
April 14, 2004

Please answer the following questions. Show all work and write neatly. Answers without justifying work will receive no credit. Partial credit will be given as appropriate, do not leave any problem blank. The point values of problems are indicated in parentheses.

1. The Hermite ordinary differential equation is frequently encountered in quantum mechanics.

$$y'' - 2xy' + 4y = 0$$

   (a) (8 points) Assuming there is a power series solution to the Hermite equation, find the recurrence relation which generates the coefficients in the power series.

(b) (6 points) Find at least the first four nonzero terms (unless the series terminates sooner) of two linearly independent series solutions to the Hermite equation.

2. (12 points) For the following ordinary differential equation, find all the singular points and determine which ones are regular singular points and which are irregular singular points.

$$(1-x)x^3y'' - (\sin x)y' + (1+x^2)y = 0$$

3. (5 points each) Consider the following ordinary differential equation.

$$(x^2 - 2x - 3)y'' + xy' + 4y = 0$$

For each of the following ordinary points determine the minimum radius of convergence for a power series solution centered at the ordinary point.

(a) $x_0 = 0$

(b) $x_0 = 1$

(c) $x_0 = 2$

4. (13 points) Use Taylor's Method (repeated differentiation) to find the first four nonzero terms of the power series solution centered at $x_0 = 0$ to the initial value problem

$$\begin{aligned} y'' + xy' + (\sin x)y &= 0 \\ y(0) &= 1 \\ y'(0) &= 0 \end{aligned}$$

5. (10 points each) Find the general solution of the following Euler equations.

(a) $x^2y'' + 3xy' + 5y = 0$

(b) $x^2y'' - 3xy' + 4y = 0$

(c) $x^2y'' - 5xy' + 9y = 0$

6. (8 points each) Consider the ordinary differential equation,

$$xy'' + 2xy' + 6e^x y = 0.$$

(a) Find the indicial equation associated with the regular singular point $x_0 = 0$.

(b) Find the exponents of singularity.