Millersville University **Name** _____

Department of Mathematics

MATH 365, *Ordinary Differential Equations*, Test 2

March 20, 2009

Please answer the following questions. Show all work and write neatly. Answers without justifying work will receive no credit. Partial credit will be given as appropriate, do not leave any problem blank. The point values of problems are indicated in parentheses.

1. (12 points) Find the Wronskian of two solutions of Legendre's equation given below.

$$(1 - t^2)y'' - 2t\,y' + \alpha(\alpha + 1)y = 0$$

2. (6 points) A spring-mass system is setup with a mass of 5 kg which stretched the spring 0.20 m. If the system is critically damped, what is the coefficient of damping γ?

3. (12 points) Find the general solution to the ordinary differential equation below.

$$9y'' + 9y' - 4y = 0$$

4. (16 points) Find the general solution to the ordinary differential equation below.

$$y'' + 9y = 4 \csc 3t$$

You may assume $0 < t < \pi/3$.

5. (14 points) Find the solution to the initial value problem below.

$$\begin{aligned} y'' + 2y' + 2y &= 0 \\ y(\pi/4) &= 0 \\ y'(\pi/4) &= -1 \end{aligned}$$

6. (12 points) Consider the ordinary differential equation

$$t^2y'' - t(t+2)y' + (t+2)y = 0$$

for which $y_1(t) = t$ is a solution. Use the method of reduction of order to find a second linearly independent solution to the ODE.

7. (16 points) Find the general solution to the ordinary differential equation below.

$$y'' + 2y' + y = 2t + \sin(2t)$$

8. (6 points each) A spring-mass system is described by the ordinary differential equation

$$u'' + \gamma u' + u = 0.$$

(a) What is the period of the undamped solution to this ODE?

(b) Find the value of γ for which the quasi period of the damped system is 3π.