

Millersville University
Department of Mathematics
MATH 365, *Ordinary Differential Equations*, Test 3
April 17, 2009

Name _____

Please answer the following questions. Show all work and write neatly. Answers without justifying work will receive no credit. Partial credit will be given as appropriate, do not leave any problem blank. The point values of problems are indicated in parentheses.

1. (18 points) Find the recurrence relation for a power series solution to

$$(1 - x)y'' - xy' + y = 0$$

near the ordinary point $x_0 = 0$.

2. (16 points) For the following ordinary differential equation, $x_0 = 0$ is a regular singular point. Find the exponents of singularity corresponding to this regular singular point. You do not need to solve the ODE.

$$x^2 y'' + 3(\sin x)y' + y = 0$$

3. (16 points) For the following ordinary differential equation find all the singular points and determine which are regular and which are irregular. You do not need to solve the ODE.

$$(\cos x)y'' + \left(\frac{1}{x - \frac{\pi}{2}}\right)y' + y = 0$$

4. (16 points) Solve the following initial value problem.

$$x^2y'' - 3xy' + 4y = 0$$

$$y(1) = 2$$

$$y'(1) = -3$$

5. (17 points) For the ordinary differential equation

$$(2x^2 + 4x)y'' + y' - xy = 0$$

$x_0 = 0$ is a regular singular point with indicial equation

$$0 = r^2 - \frac{3}{4}r.$$

Find the recurrence relation for a series solution centered at $x_0 = 0$ to this ODE corresponding to the larger of the two exponents of singularity.

6. (17 points) Solve the following ordinary differential equation.

$$x^2 y'' + xy' + 4y = \sin(\ln x)$$