The completed assignment is due at class time on 10/23/2003. You may use your textbook, computer programs, and notes. All numerical approximations must be accurate to within $10^{-4}$ unless otherwise stated.

Star S in the Big Dipper (Ursa Major) has a regular variation in its apparent magnitude. Leon Campbell and Laizi Jacchia give data for the mean light curve of this star in the book *The Story of Variable Stars* (1941). The data are shown in the table below.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>−110</td>
<td>7.98</td>
</tr>
<tr>
<td>−80</td>
<td>8.85</td>
</tr>
<tr>
<td>−40</td>
<td>10.71</td>
</tr>
<tr>
<td>−10</td>
<td>11.70</td>
</tr>
<tr>
<td>30</td>
<td>10.01</td>
</tr>
<tr>
<td>80</td>
<td>8.23</td>
</tr>
<tr>
<td>110</td>
<td>7.86</td>
</tr>
</tbody>
</table>

What is different about this data is that the apparent magnitude of the star is periodic, in other words the magnitude at phase −120 is the same as the magnitude at phase 120.

1. Derive a linear system to find the coefficients of a cubic spline interpolant with periodic boundary conditions. For periodic boundary conditions the first and second derivatives agree at the boundary.

2. Solve the linear system (using a computer if necessary) to find the appropriate cubic spline interpolant for the data.

3. What is the error in the cubic spline interpolant if the following additional data are available for star S?

<table>
<thead>
<tr>
<th>Phase</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>−100</td>
<td>8.37</td>
</tr>
<tr>
<td>−60</td>
<td>9.40</td>
</tr>
<tr>
<td>−20</td>
<td>11.39</td>
</tr>
<tr>
<td>20</td>
<td>10.84</td>
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<tr>
<td>60</td>
<td>8.53</td>
</tr>
<tr>
<td>100</td>
<td>7.89</td>
</tr>
</tbody>
</table>