Please work the following problem for homework and turn it in at class time on Tuesday, February 14, 2006. A successfully completed assignment is worth 20 points.

Use the method of separation of variables to find a steady state solution to the two-dimensional heat equation

$$u_t = u_{xx} + u_{yy}$$

on the rectangle where $0 < x < 2$ and $0 < y < 1$. The following boundary conditions are in effect:

- $u(0, y) = 0$ for $0 < y < 1$,
- $u(2, y) = \sin \frac{\pi y}{2}$ for $0 < y < 1$,
- $u(x, 0) = 0$ for $0 < x < 2$,
- $u(x, 1) = 1 + \sin 2\pi x$ for $0 < x < 2$. 