Please work the following problems for homework and turn them in at class time on Tuesday, March 21, 2006. Each problem is worth 10 points unless marked otherwise.

1. Consider an elastic string of length $L = 1$ whose ends are held fixed for all $t$. The string is given an initial displacement from equilibrium described by the function $f(x)$ and is released with no initial velocity. If the wave speed of the string is $c = 1$, find the displacement of the string as a function of $x$ and $t$. Plot the displacement of the string for $t = 0, 0.2, 0.4, 0.6, 0.8, 1$.

$$f(x) = 8x(1-x)^2$$

2. Consider an elastic string of length $L = 1$ whose ends are held fixed for all $t$. The string is given an initial velocity described by the function $g(x)$ and is released from the equilibrium position. If the wave speed of the string is $c = 1$, find the displacement of the string as a function of $x$ and $t$. Plot the displacement of the string for $t = 0, 0.1, 0.2, \ldots, 2$.

$$g(x) = \begin{cases} 
0 & \text{if } 0 \leq x < 1/4 \\
1 & \text{if } 1/4 \leq x < 3/4 \\
0 & \text{if } 3/4 \leq x < 1 
\end{cases}$$

3. Find the solution of the wave equation

$$u_{tt} = c^2 u_{xx} \quad \text{for } 0 < x < 1, \ t > 0$$

$$u(0,t) = u(1,t) = 0 \quad \text{for } t > 0$$

$$u(x,0) = \sin \pi x \quad \text{for } 0 < x < 1$$

$$u_t(x,0) = c \cos \pi x \quad \text{for } 0 < x < 1.$$