

Millersville University

Department of Mathematics

MATH 467 *Partial Differential Equations*, Miscellaneous Wave Equation Problems

March 29, 2006

Find the solution to the following initial boundary value problems for the wave equation. Express the solution in the form of

$$u(x, t) = \frac{1}{2} (\hat{f}(x + ct) + \hat{f}(x - ct)) + \frac{1}{2} \int_{x-ct}^{x+ct} \hat{g}(z) dz.$$

### Example 1

$$\begin{aligned} u_{tt} &= u_{xx} \quad \text{for } 0 < x < 1, t > 0 \\ u(0, t) &= u(1, t) = 0 \quad \text{for } t > 0 \\ u(x, 0) &= \begin{cases} 2x & \text{if } 0 \leq x \leq \frac{1}{2} \\ 2(1-x) & \text{if } \frac{1}{2} \leq x \leq 1 \end{cases} \\ u_t(x, 0) &= 0 \quad \text{for } 0 \leq x \leq 1 \end{aligned}$$

### Example 2

$$\begin{aligned} u_{tt} &= u_{xx} \quad \text{for } 0 < x < 1, t > 0 \\ u(0, t) &= u(1, t) = 0 \quad \text{for } t > 0 \\ u(x, 0) &= 0 \quad \text{for } 0 \leq x \leq 1 \\ u_t(x, 0) &= \begin{cases} 4x & \text{if } 0 \leq x \leq \frac{1}{4} \\ 1 & \text{if } \frac{1}{4} \leq x \leq \frac{3}{4} \\ 4(1-x) & \text{if } \frac{3}{4} \leq x \leq 1 \end{cases} \end{aligned}$$

### Example 3

$$\begin{aligned} u_{tt} &= u_{xx} \quad \text{for } 0 < x < 4, t > 0 \\ u(0, t) &= u(4, t) = 0 \quad \text{for } t > 0 \\ u(x, 0) &= \begin{cases} 1 & \text{if } 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases} \\ u_t(x, 0) &= x(4-x) \quad \text{for } 0 \leq x \leq 4 \end{aligned}$$